Coupled-Ladder Reflectionless Filters

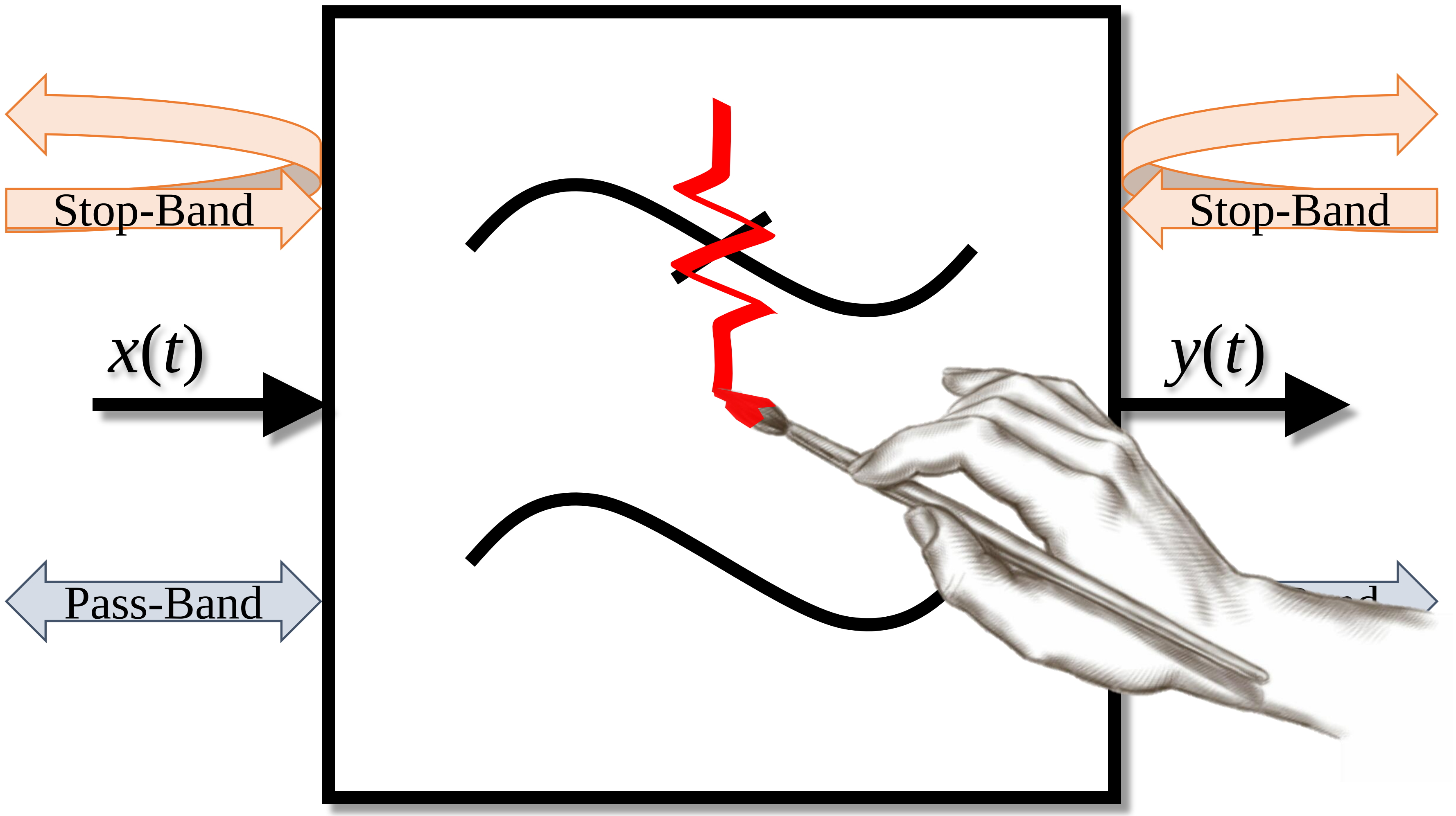
Latin America Microwave Conference, San Jose

12/2023

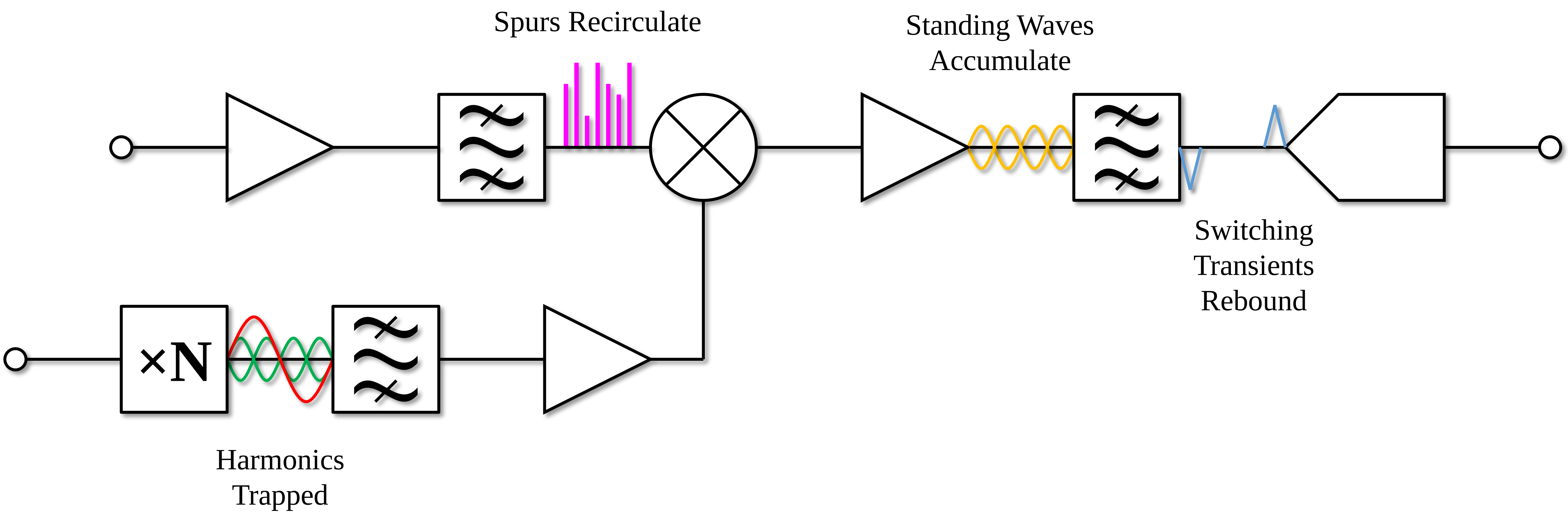
Matt Morgan, NRAO

Reflectionless Filters

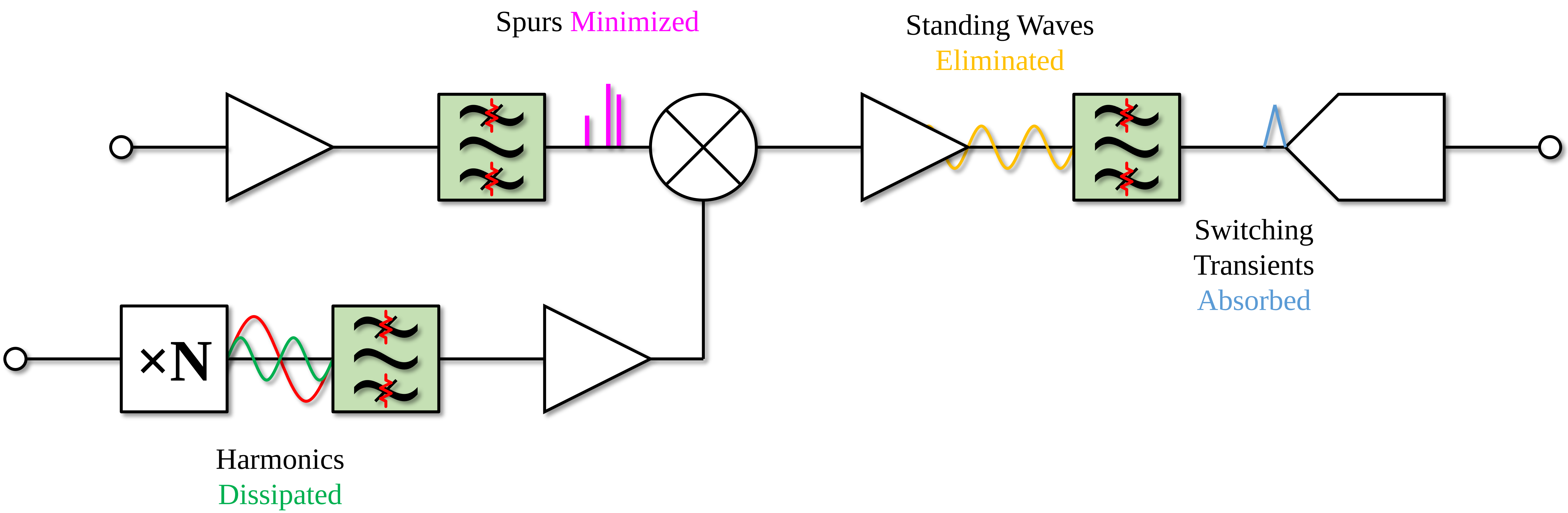
What is a Reflectionless Filter?



Issues with Conventional Filters

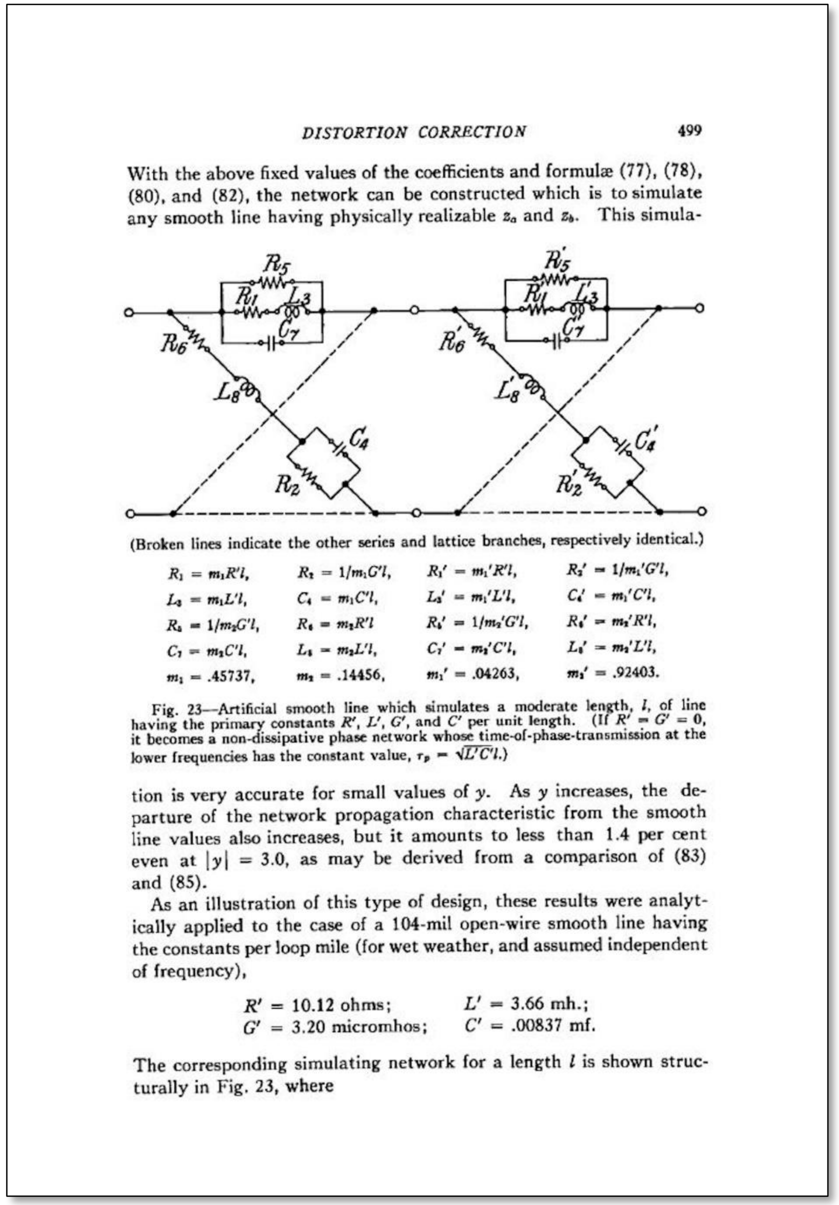


Benefits of Reflectionless Filters



Once Called *Constant-Resistance Networks*

Otto Zobel, 1928



DISTORTION CORRECTION 499

With the above fixed values of the coefficients and formulæ (77), (78), (80), and (82), the network can be constructed which is to simulate any smooth line having physically realizable z_a and z_b. This simula-

(Broken lines indicate the other series and lattice branches, respectively identical.)

$R_1 = m_1R'l$, $R_2 = 1/m_1G'l$, $R_1' = m_1'R'l$, $R_2' = 1/m_1'G'l$,
$L_3 = m_1L'l$, $C_4 = m_1C'l$, $L_3' = m_1'L'l$, $C_4' = m_1'C'l$,
$R_5 = 1/m_2G'l$, $R_6 = m_2R'l$ $R_5' = 1/m_2'G'l$, $R_6' = m_2'R'l$,
$C_7 = m_2C'l$, $L_8 = m_2L'l$, $C_7' = m_2'C'l$, $L_8' = m_2'L'l$,
$m_1 = .45737$, $m_2 = .14456$, $m_1' = .04263$, $m_2' = .92403$.

Fig. 23—Artificial smooth line which simulates a moderate length, l, of line having the primary constants R', L', G', and C' per unit length. (If $R' = G' = 0$, it becomes a non-dissipative phase network whose time-of-phase-transmission at the lower frequencies has the constant value, $\tau_p = \sqrt{L'C'}l$.)

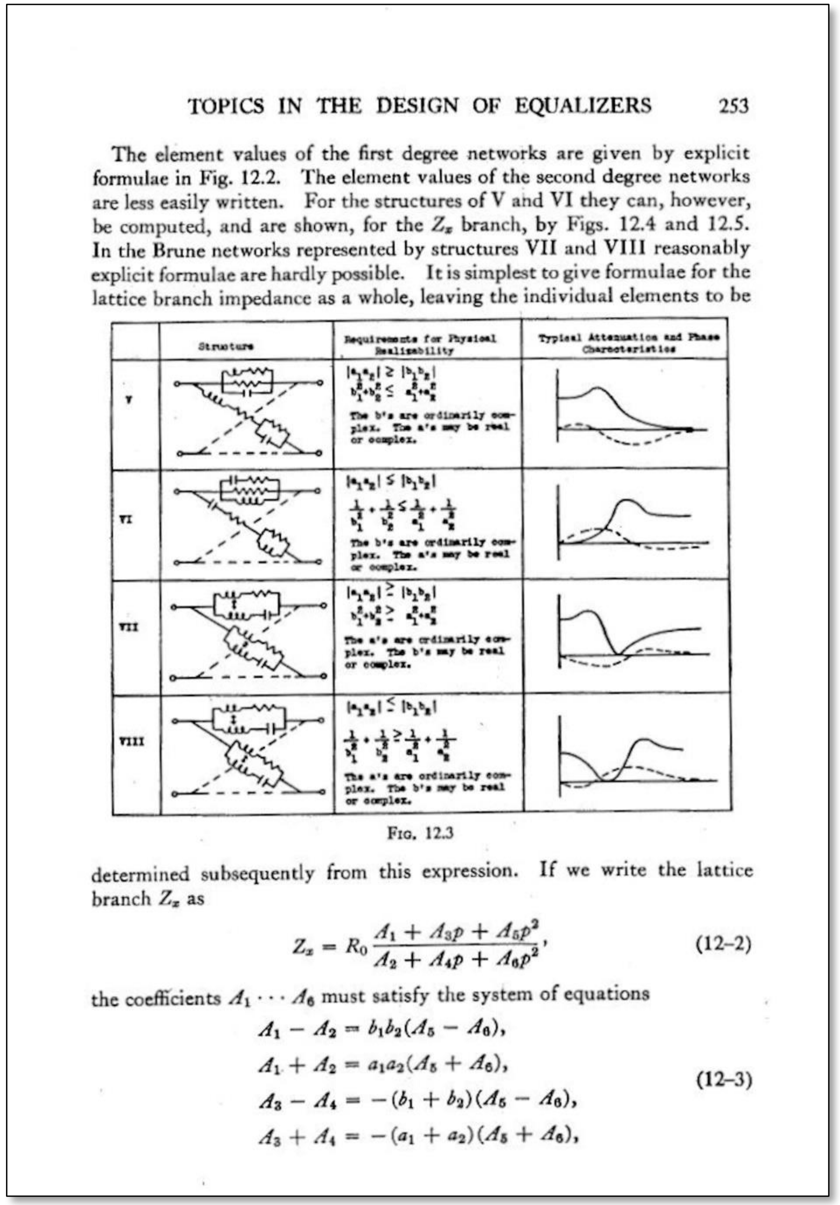
tion is very accurate for small values of y. As y increases, the departure of the network propagation characteristic from the smooth line values also increases, but it amounts to less than 1.4 per cent even at $|y| = 3.0$, as may be derived from a comparison of (83) and (85).

As an illustration of this type of design, these results were analytically applied to the case of a 104-mil open-wire smooth line having the constants per loop mile (for wet weather, and assumed independent of frequency),

$$R' = 10.12 \text{ ohms}; \quad L' = 3.66 \text{ mh.};$$
$$G' = 3.20 \text{ micromhos}; \quad C' = .00837 \text{ mf.}$$

The corresponding simulating network for a length l is shown structurally in Fig. 23, where

Hendrik Bode, 1945



TOPICS IN THE DESIGN OF EQUALIZERS 253

The element values of the first degree networks are given by explicit formulae in Fig. 12.2. The element values of the second degree networks are less easily written. For the structures of V and VI they can, however, be computed, and are shown, for the Z_x branch, by Figs. 12.4 and 12.5. In the Brune networks represented by structures VII and VIII reasonably explicit formulae are hardly possible. It is simplest to give formulae for the lattice branch impedance as a whole, leaving the individual elements to be

	Structure	Requirements for Physical Realizability	Typical Attenuation and Phase Characteristics
V		$\lvert a_1a_2\rvert \geq \lvert b_1b_2\rvert$ $b_1^2+b_2^2 \leq a_1^2+a_2^2$ The b's are ordinarily complex. The a's may be real or complex.	
VI		$\lvert a_1a_2\rvert \leq \lvert b_1b_2\rvert$ $\frac{1}{b_1^2}+\frac{1}{b_2^2} \leq \frac{1}{a_1^2}+\frac{1}{a_2^2}$ The b's are ordinarily complex. The a's may be real or complex.	
VII		$\lvert a_1a_2\rvert \gtreqless \lvert b_1b_2\rvert$ $b_1^2+b_2^2 \gtreqless a_1^2+a_2^2$ The a's are ordinarily complex. The b's may be real or complex.	
VIII		$\lvert a_1a_2\rvert \lesseqgtr \lvert b_1b_2\rvert$ $\frac{1}{b_1^2}+\frac{1}{b_2^2} \geq \frac{1}{a_1^2}+\frac{1}{a_2^2}$ The a's are ordinarily complex. The b's may be real or complex.	

Fig. 12.3

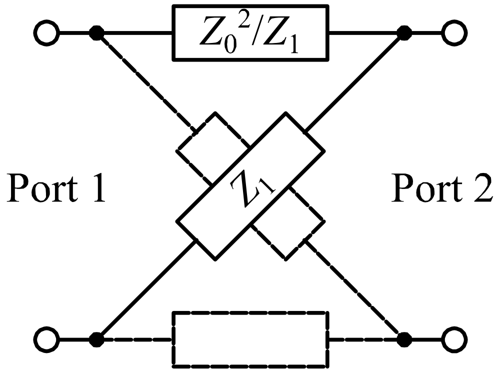
determined subsequently from this expression. If we write the lattice branch Z_x as

$$Z_x = R_0 \frac{A_1 + A_3p + A_5p^2}{A_2 + A_4p + A_6p^2}, \qquad (12\text{–}2)$$

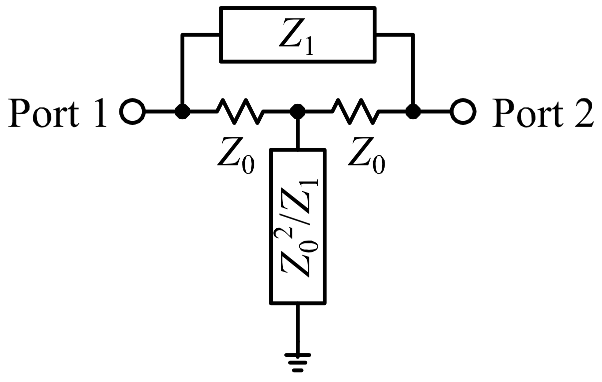
the coefficients $A_1 \cdots A_6$ must satisfy the system of equations

$$\begin{aligned} A_1 - A_2 &= b_1b_2(A_5 - A_6), \\ A_1 + A_2 &= a_1a_2(A_5 + A_6), \\ A_3 - A_4 &= -(b_1 + b_2)(A_5 - A_6), \\ A_3 + A_4 &= -(a_1 + a_2)(A_5 + A_6), \end{aligned} \qquad (12\text{–}3)$$

Core Networks:

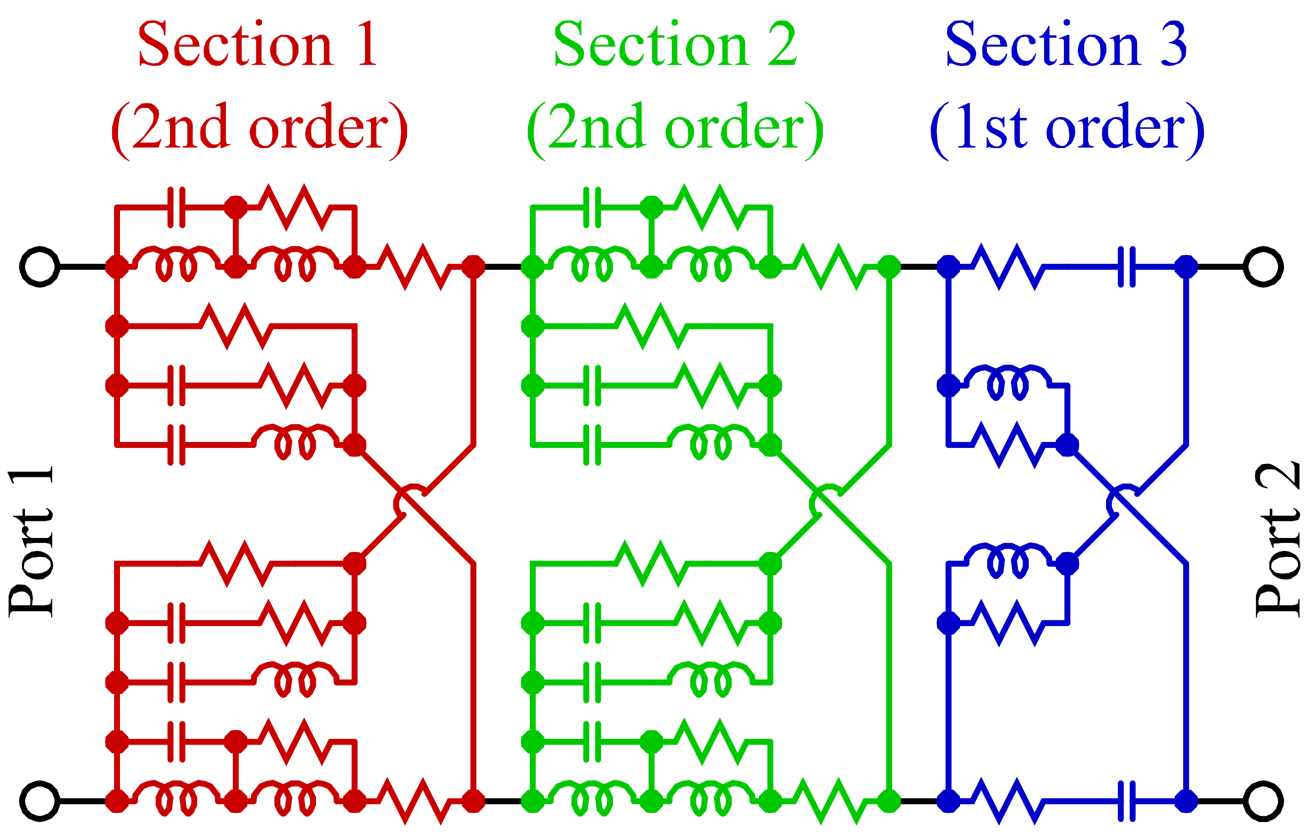


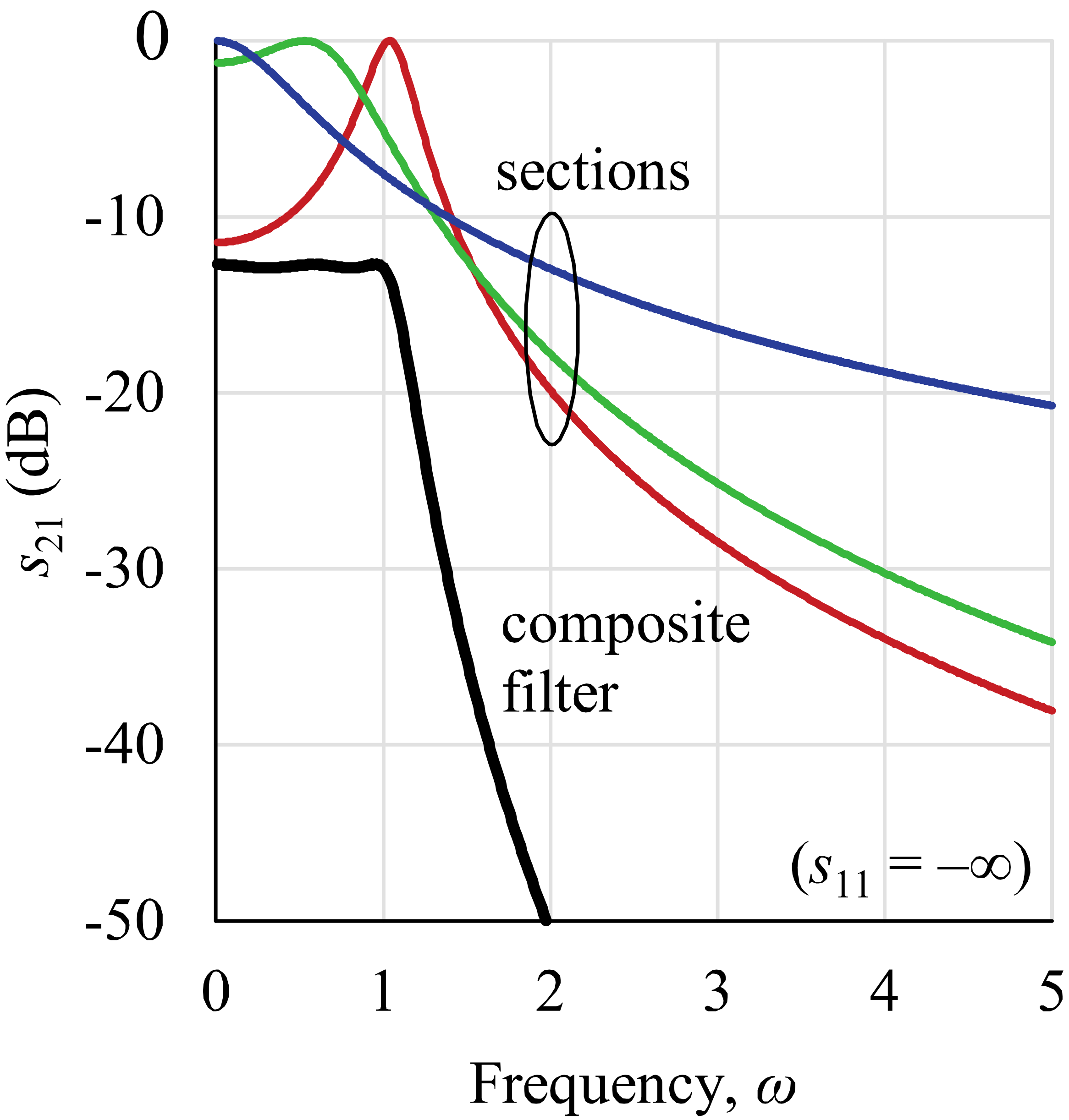
$$s_{21} = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$



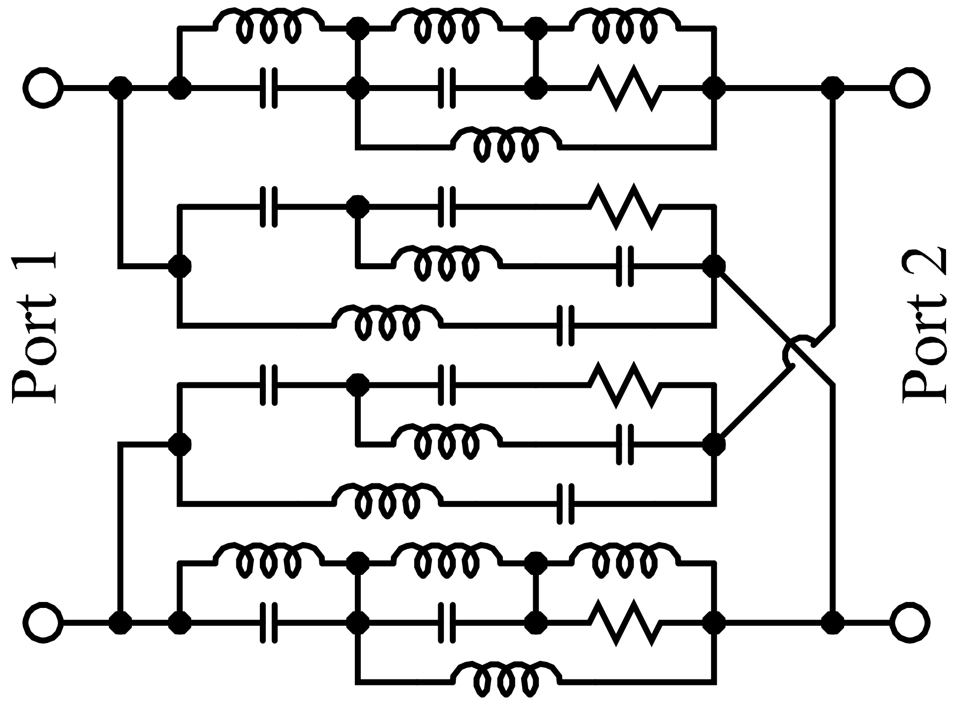
$$s_{21} = \frac{Z_0}{Z_0 + Z_1}$$

Realizability Limitations Lead to Excess Flat Loss

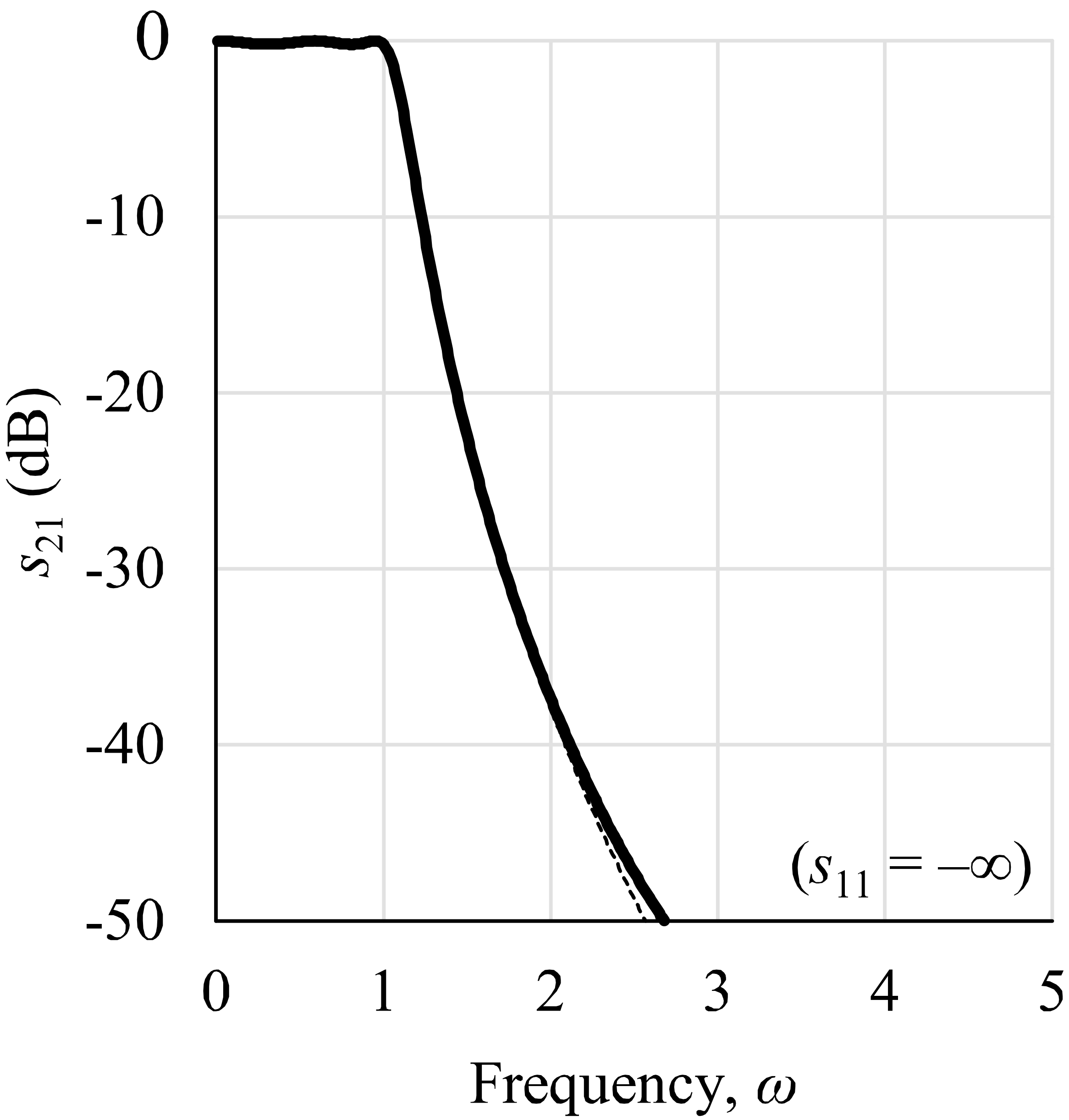




Higher-Order Synthesis Complex (High Element Count)



Fifth-Order Filter, **28 elements!**

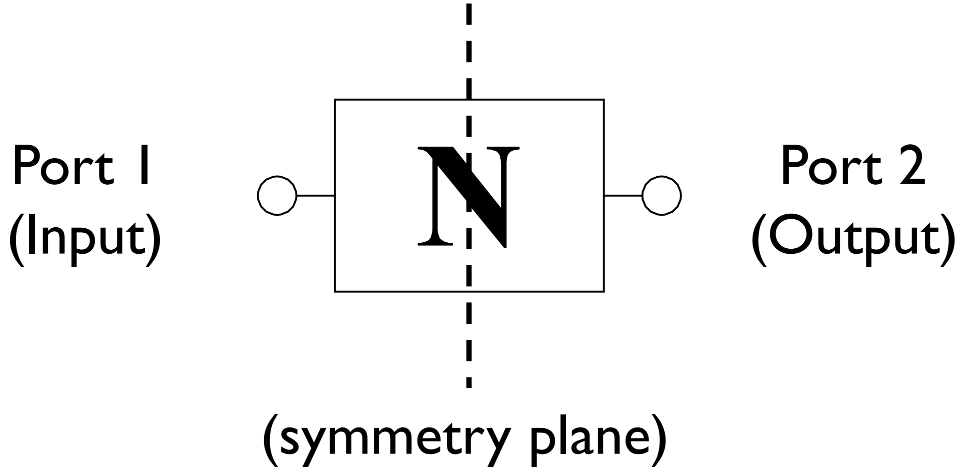


Renewed Interest

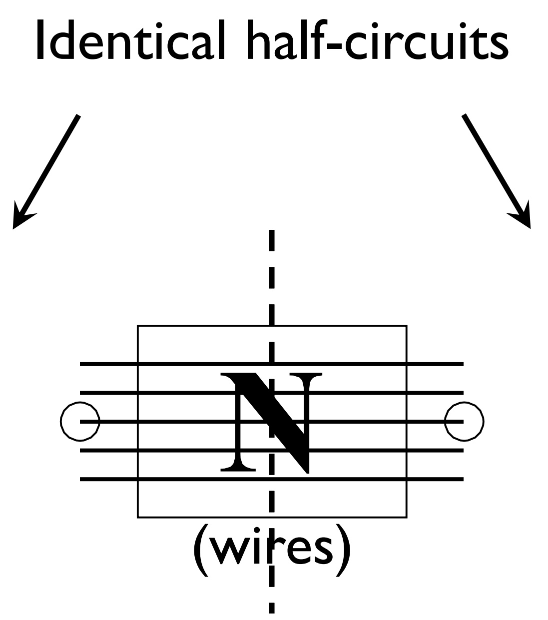
- Discovery of coupled-ladder topologies has generated renewed interest in reflectionless filters.
- Many researchers now exploring different ways of implementing reflectionless filters
  - Transmission-Lines
  - Coaxial Resonators
  - Surface-Acoustic Wave
  - Substrate-Integrated Waveguide
- Many of these new approaches are based on empirical modeling.
- For this talk, I will focus on the fundamental coupled-ladder solution.

Coupled Ladder Topology

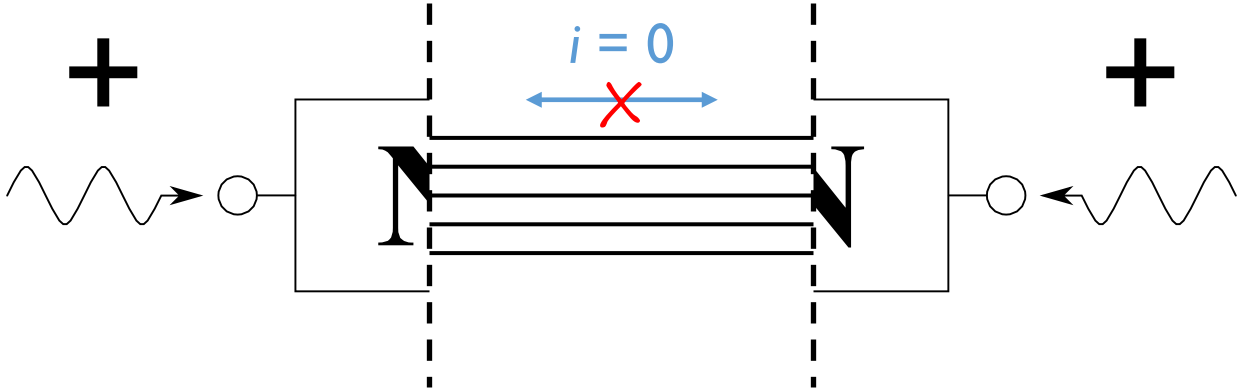
A Symmetric Two-Port Network, “N”



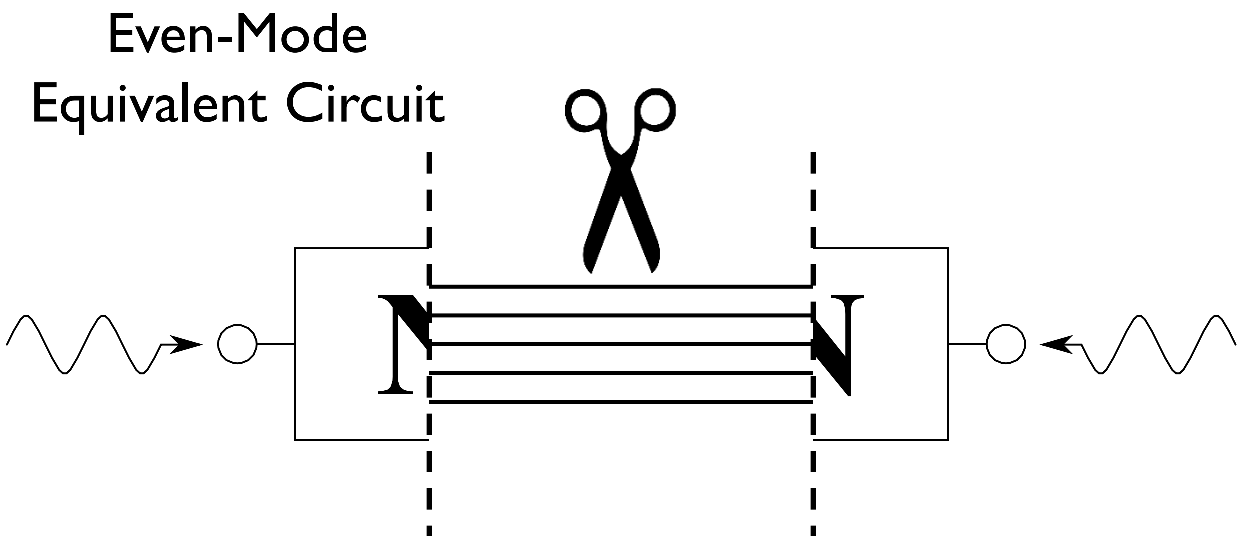
A Symmetric Two-Port Network, “N”



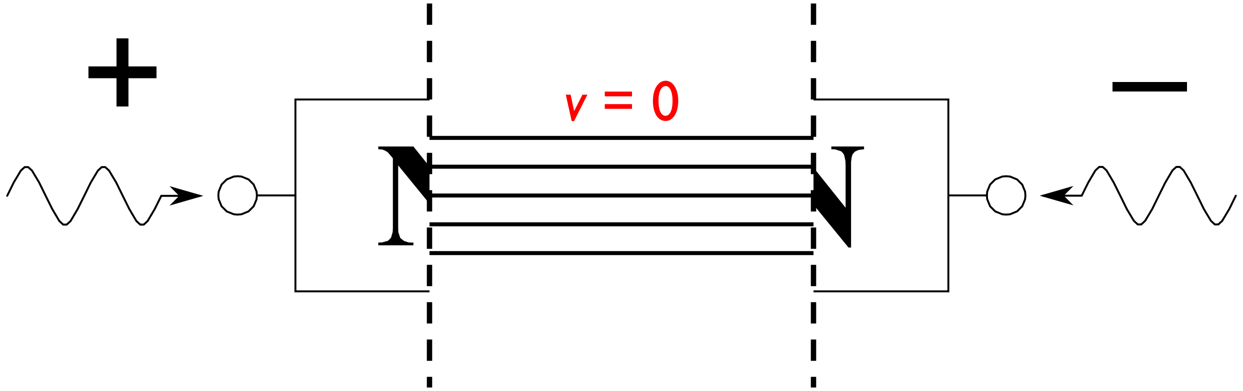
Even-Mode Excitation



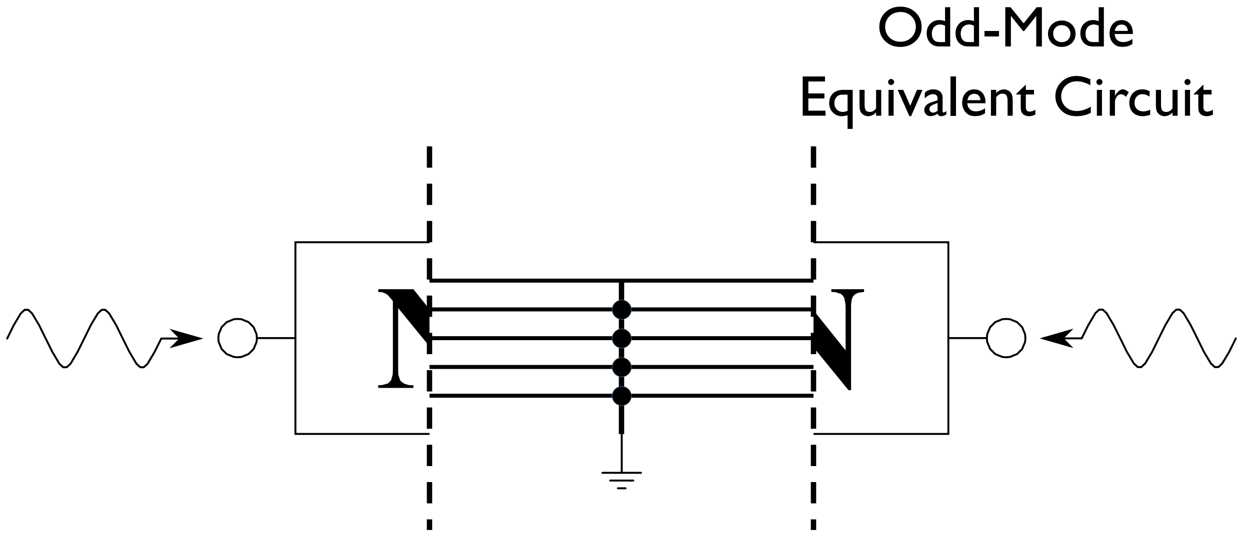
Even-Mode Excitation



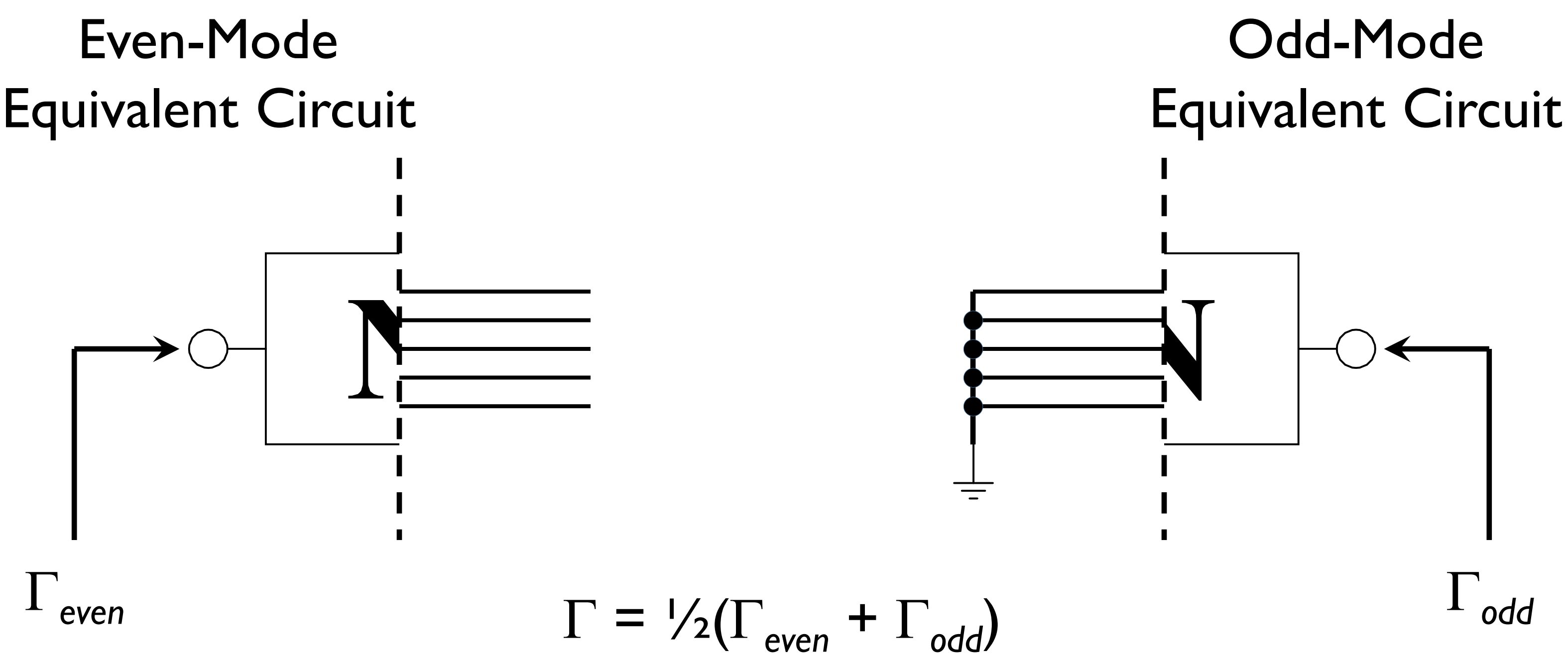
Odd-Mode Excitation



Odd-Mode Excitation



Even and Odd-Mode *Analysis*

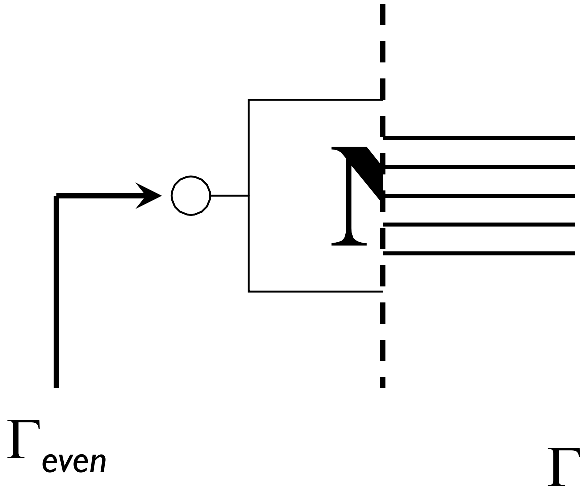


$\Gamma = ½(\Gamma_{even} + \Gamma_{odd})$

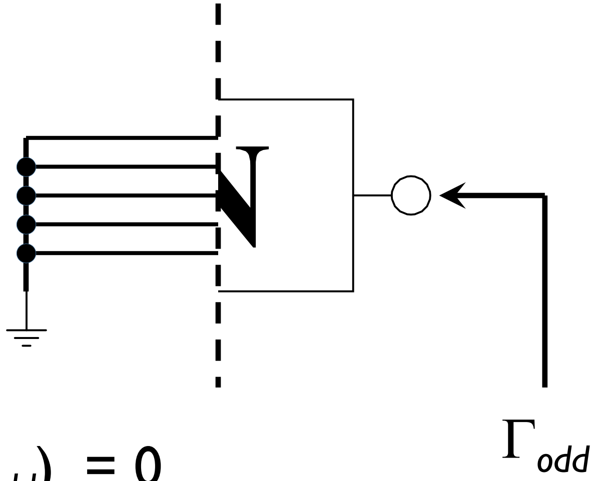
$T = ½(\Gamma_{even} - \Gamma_{odd})$

Even and Odd-Mode *Synthesis*

Even-Mode
Equivalent Circuit



Odd-Mode
Equivalent Circuit



$$\Gamma = \tfrac{1}{2}(\Gamma_{even} + \Gamma_{odd}) = 0$$

$$T = \tfrac{1}{2}(\Gamma_{even} - \Gamma_{odd}) = T(f)$$

Therefore…

$\Gamma_{even} = -\Gamma_{odd}$ *duality*

$\Gamma_{even} = T(f)$ *~high-pass transformation*

Duality

$$\Gamma_{even} = -\Gamma_{odd}$$

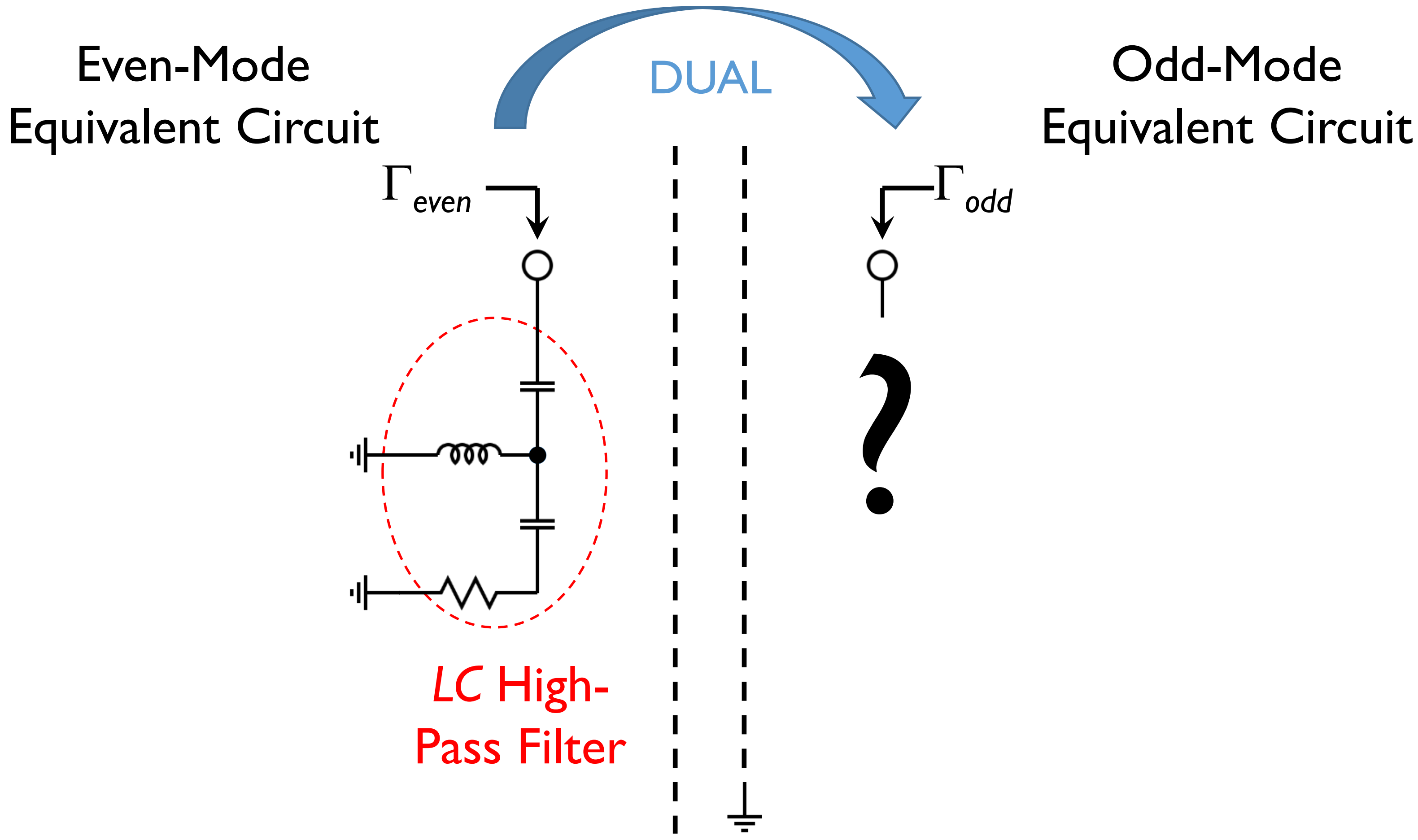
$$\frac{Z_{even} - Z_0}{Z_{even} + Z_0} = \frac{Z_0 - Z_{odd}}{Z_0 + Z_{odd}}$$

$$\frac{z_e - 1}{z_e + 1} = \frac{1 - z_o}{1 + z_o}$$

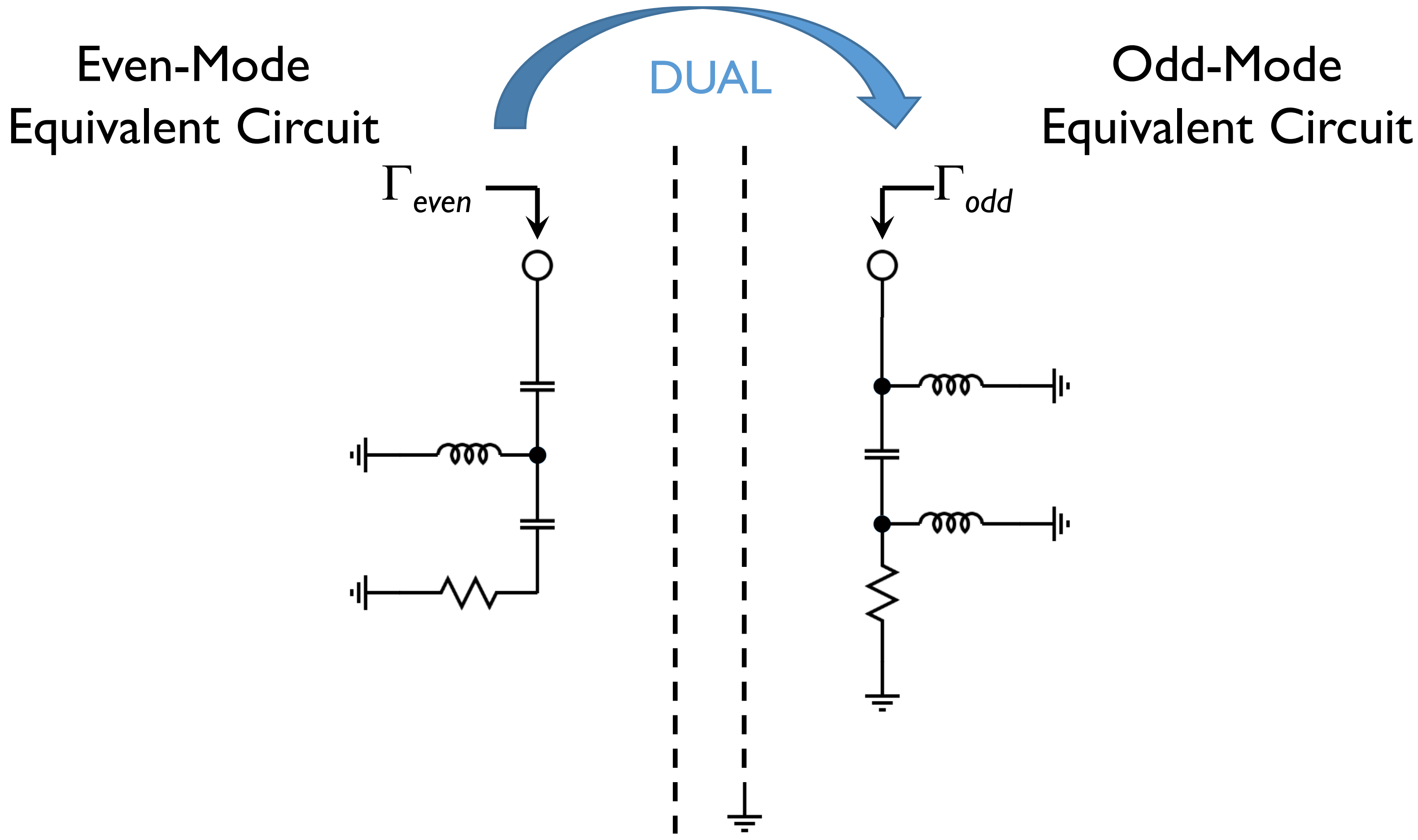
$$\frac{z_e - 1}{z_e + 1} = \frac{y_o - 1}{y_o + 1}$$

$$z_e = y_o$$

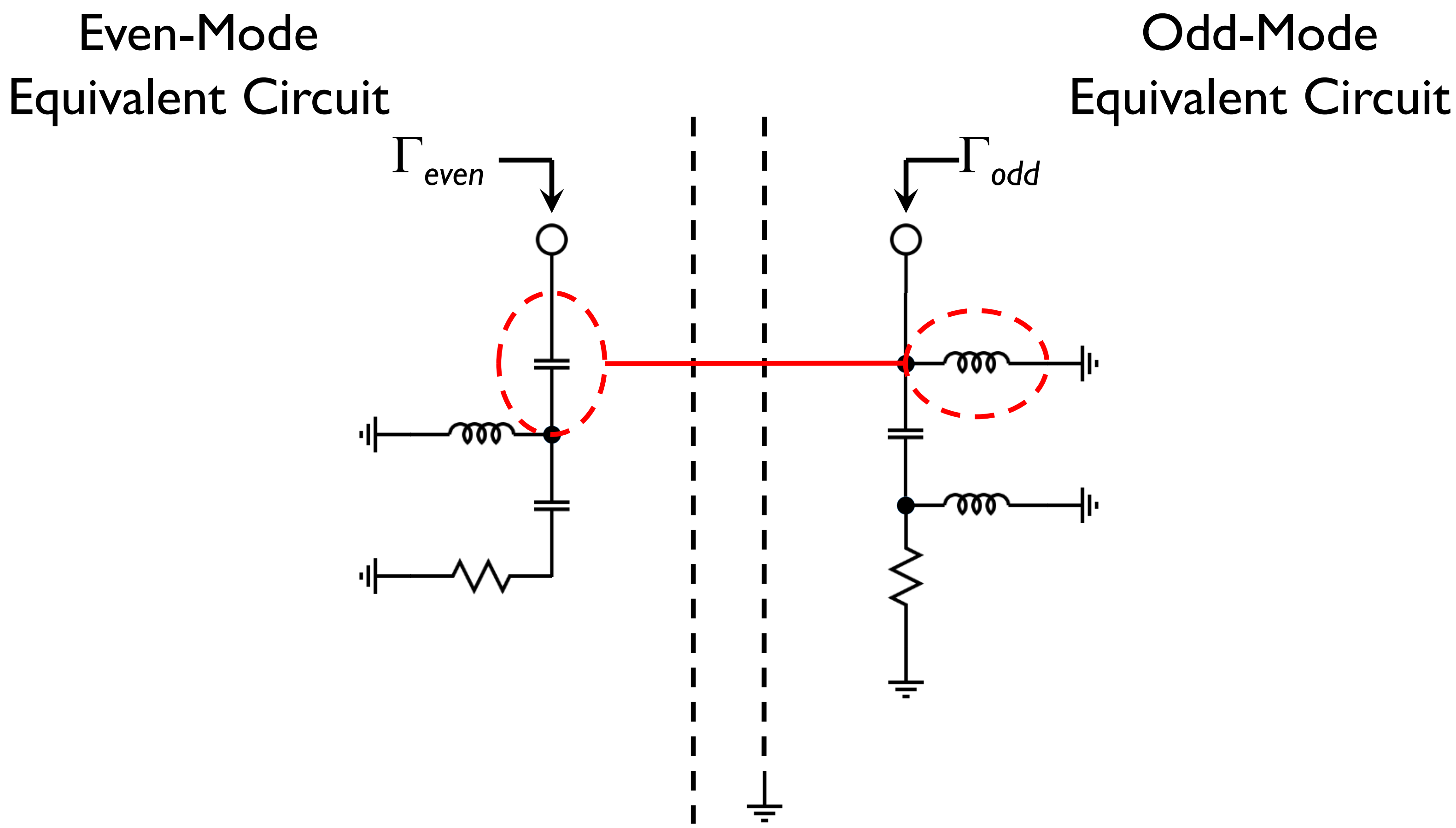
Even and Odd-Mode *Synthesis*



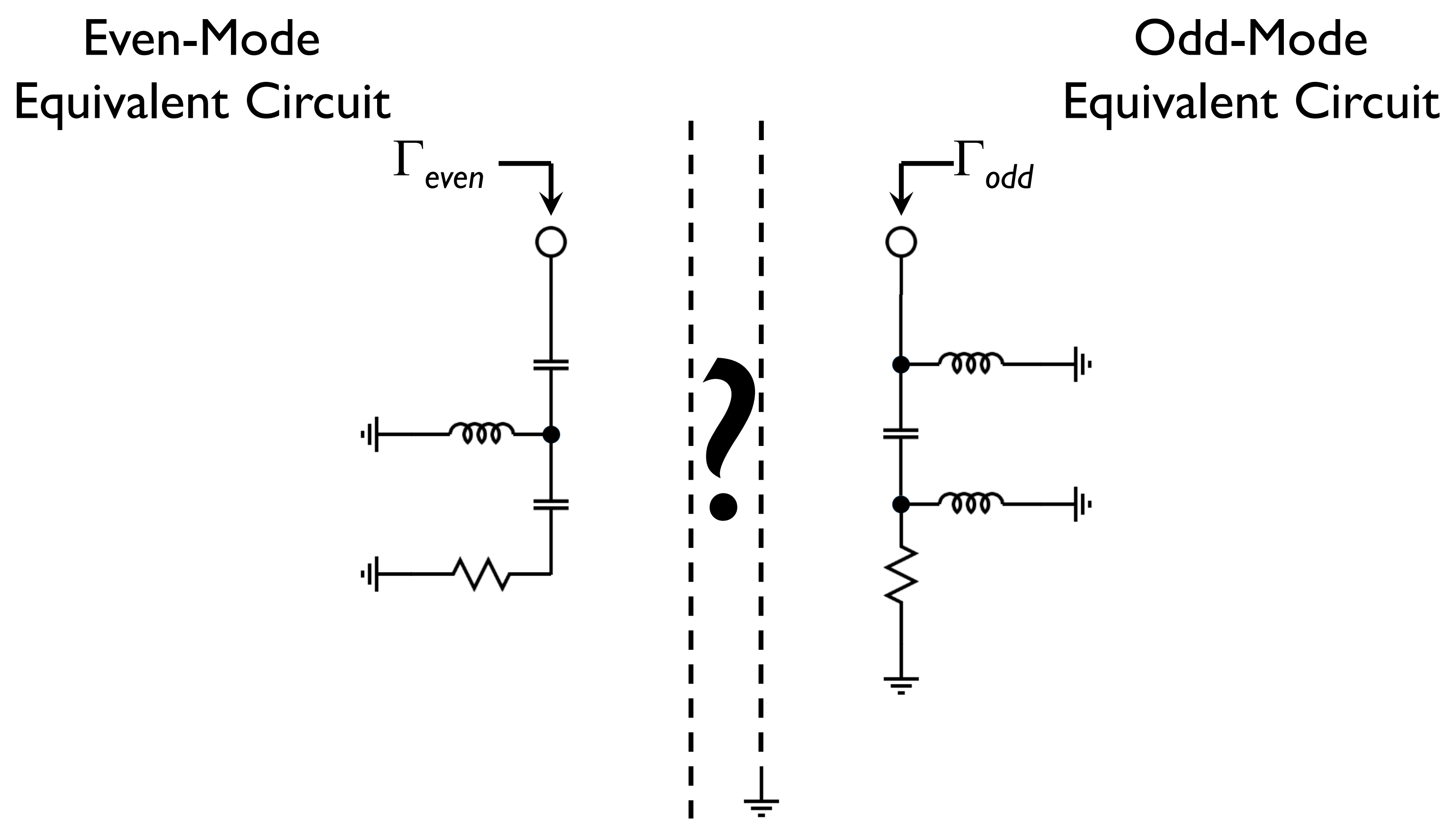
Even and Odd-Mode *Synthesis*



Even and Odd-Mode *Synthesis*



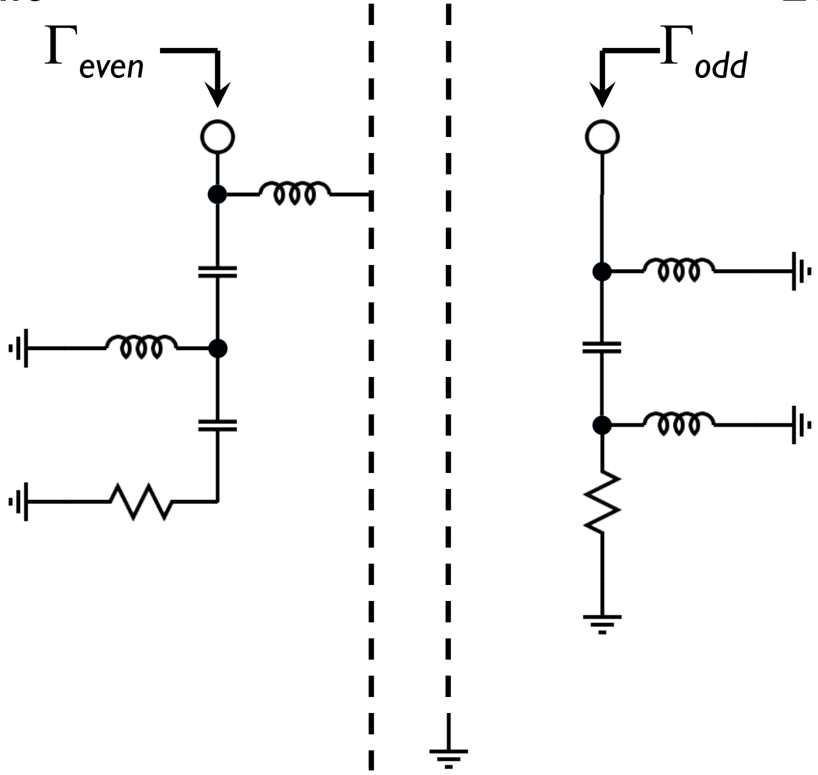
Even and Odd-Mode *Synthesis*



Even and Odd-Mode *Synthesis*

Even-Mode
Equivalent Circuit

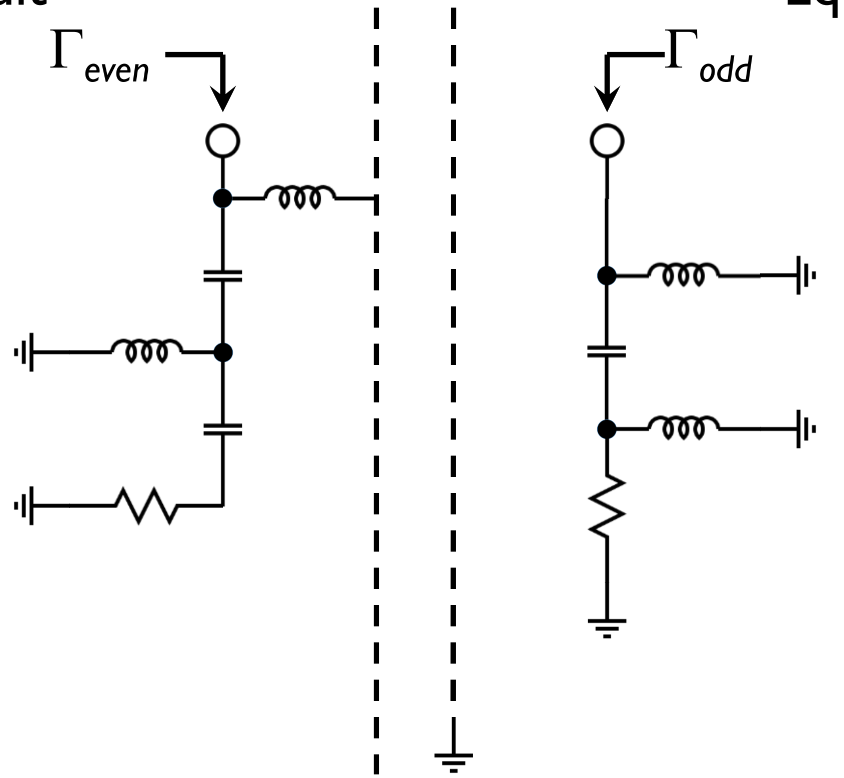
Odd-Mode
Equivalent Circuit



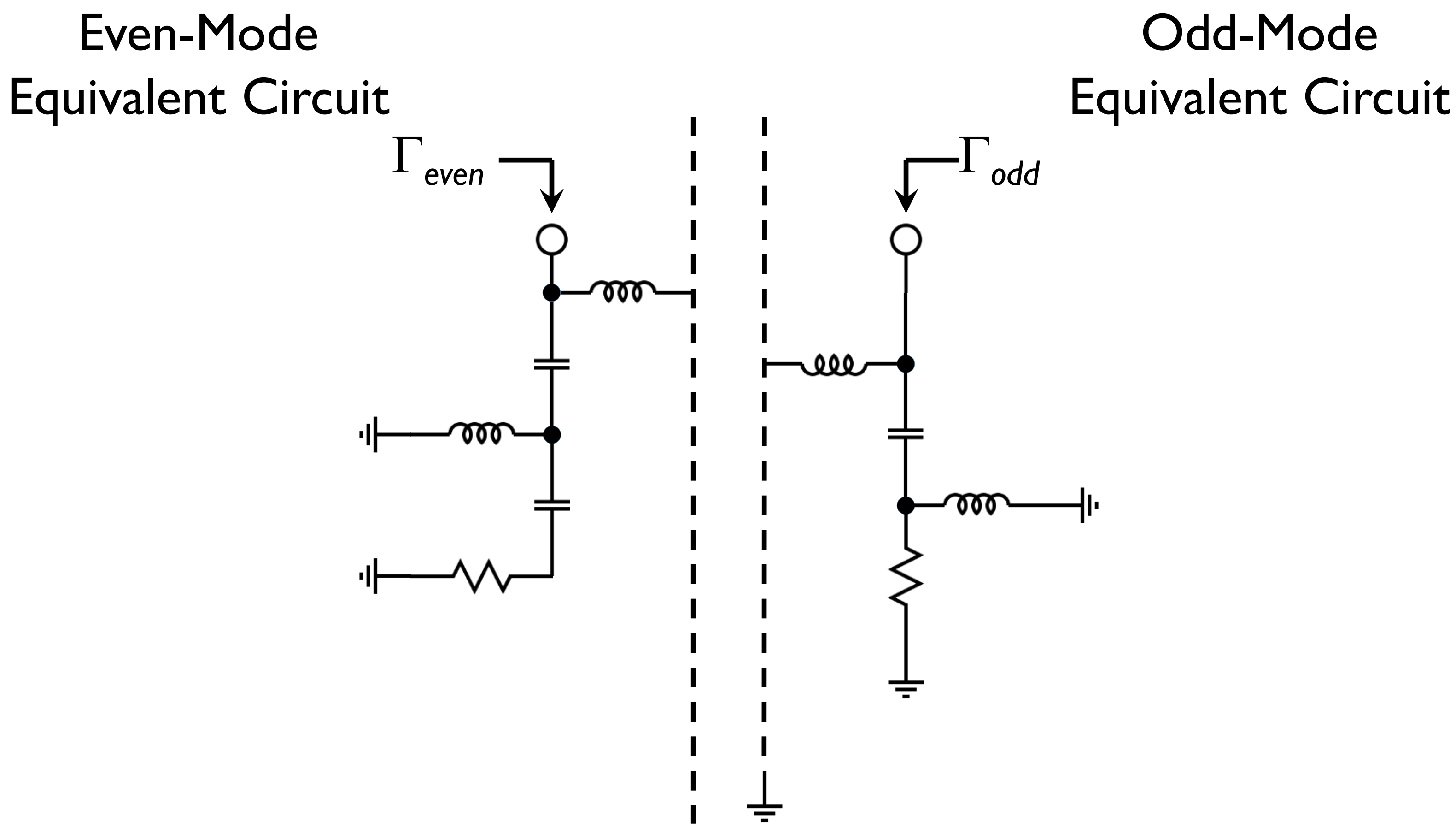
Even and Odd-Mode *Synthesis*

Even-Mode
Equivalent Circuit

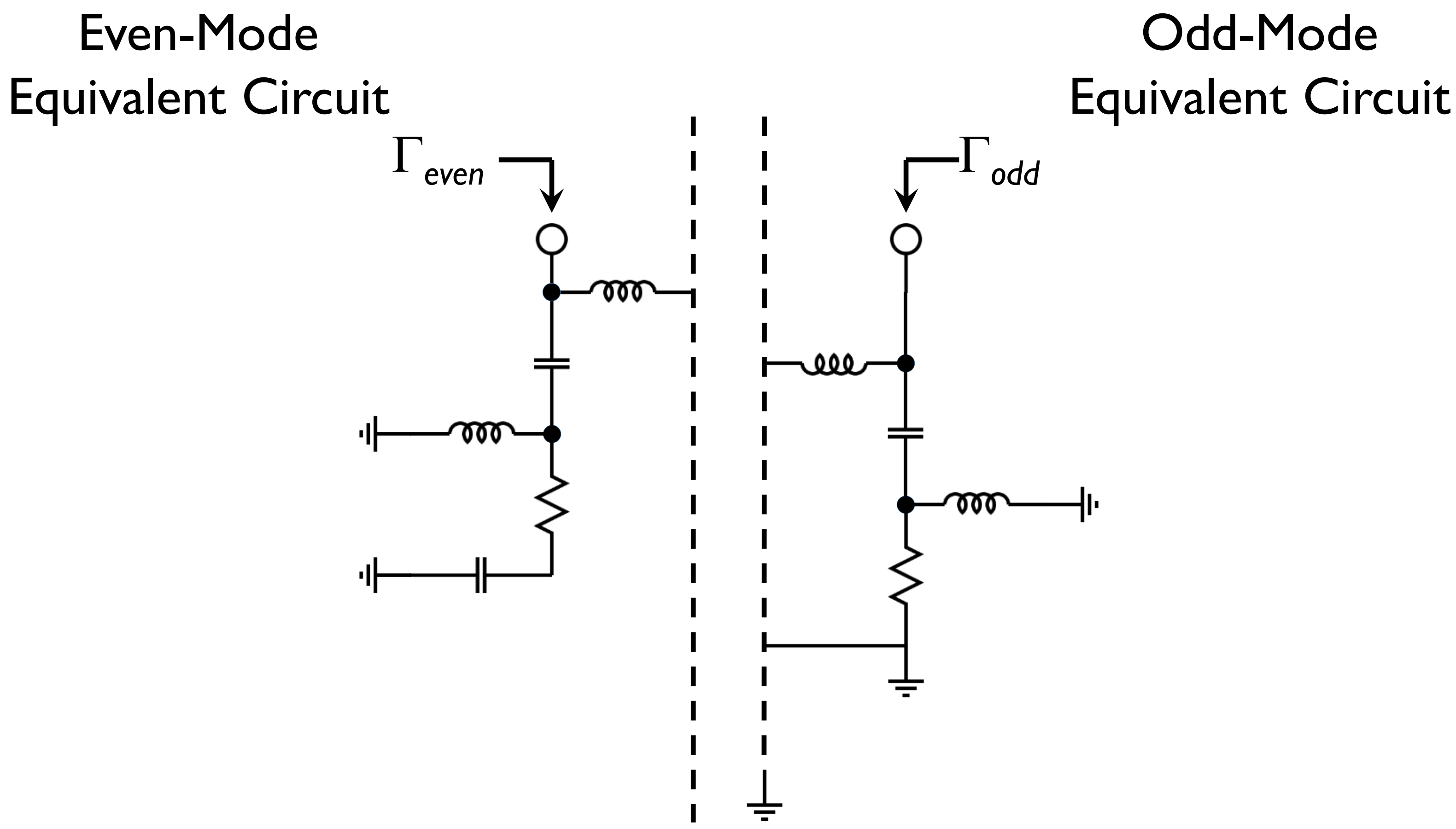
Odd-Mode
Equivalent Circuit



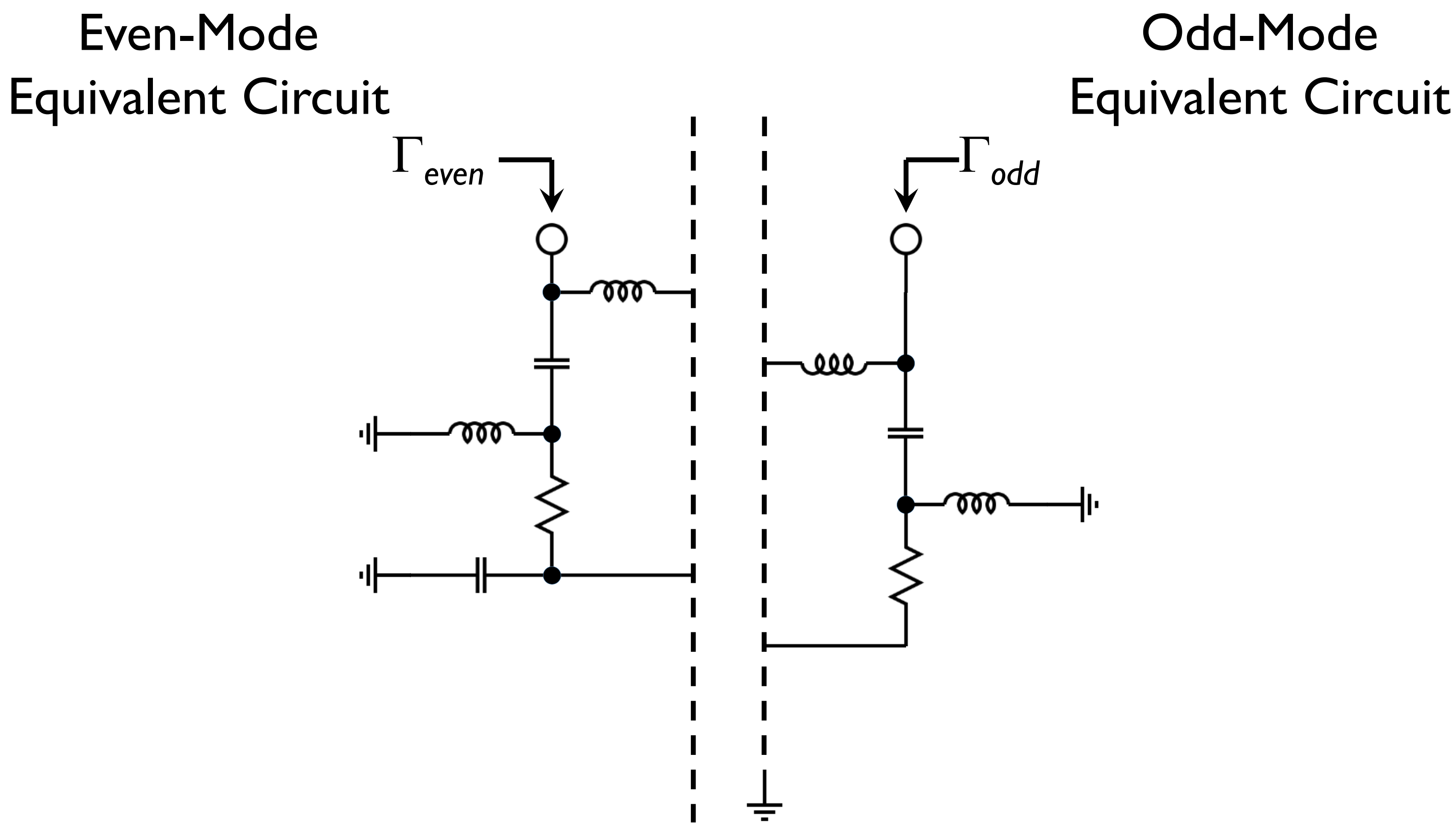
Even and Odd-Mode *Synthesis*



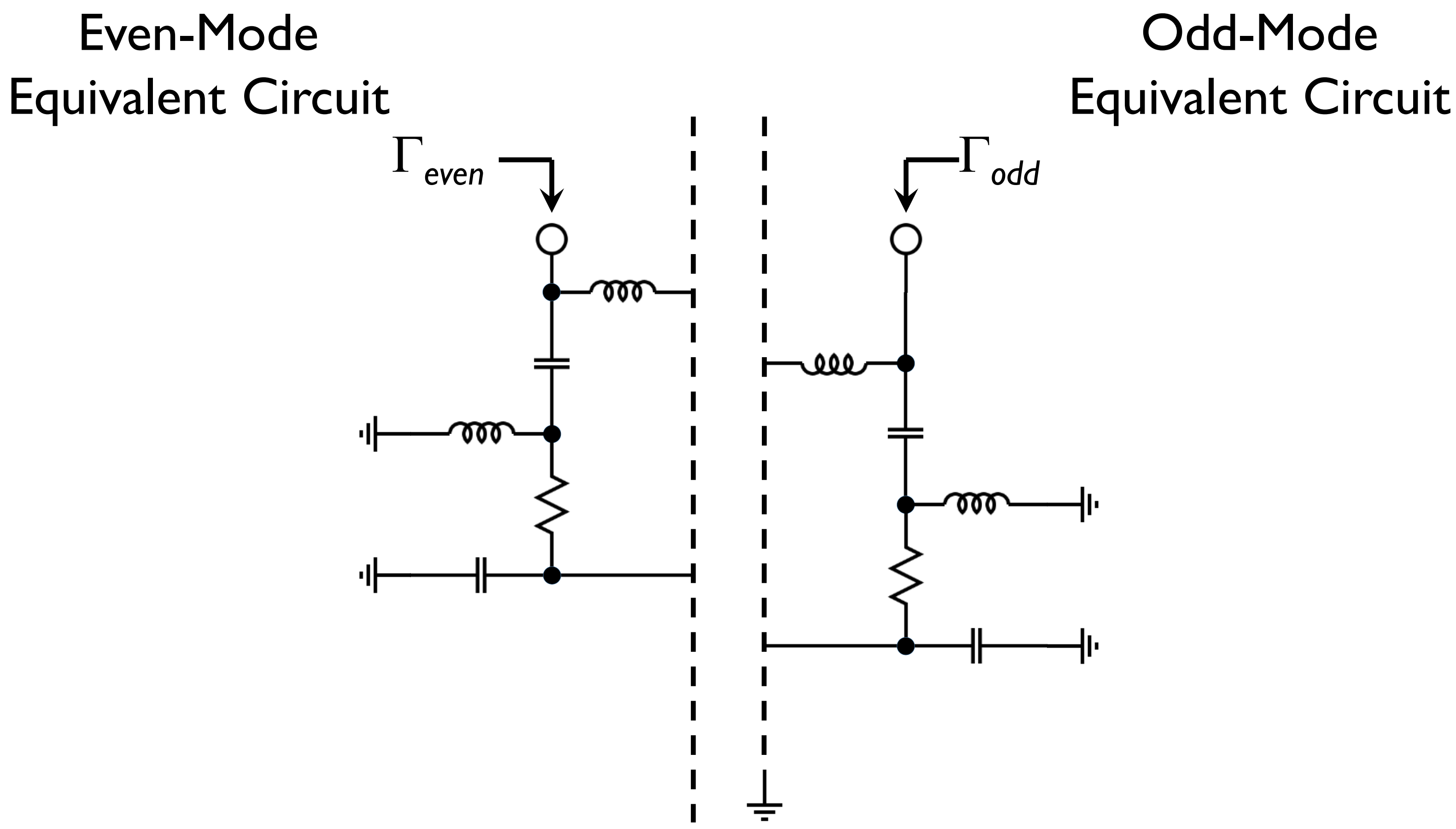
Even and Odd-Mode *Synthesis*



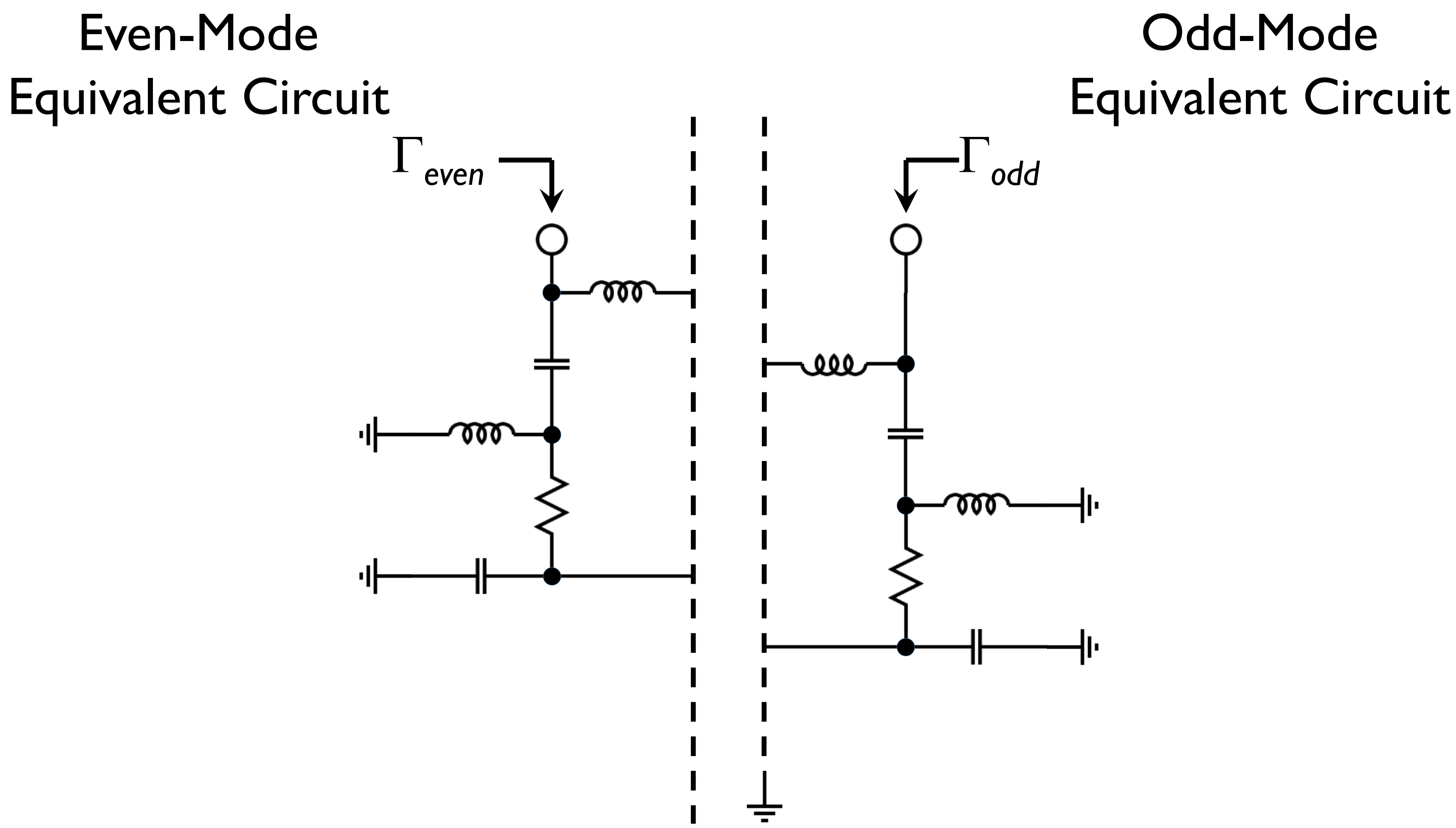
Even and Odd-Mode *Synthesis*



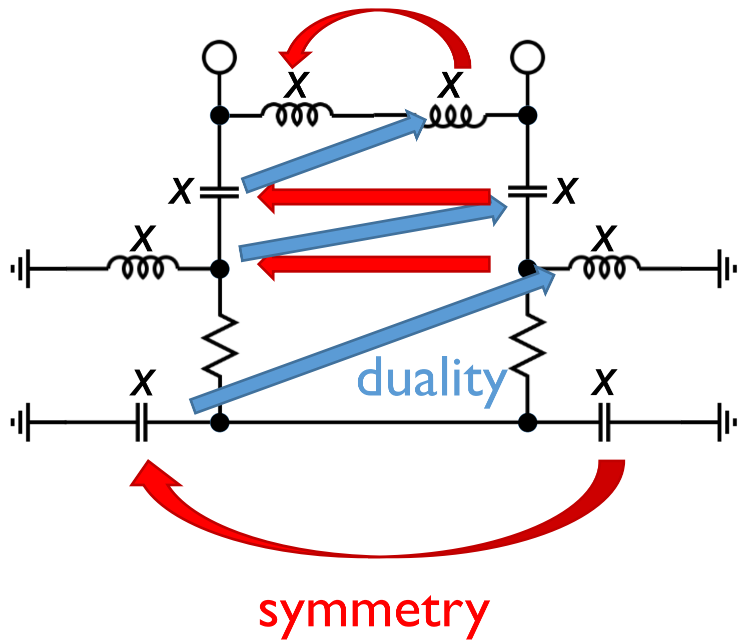
Even and Odd-Mode *Synthesis*



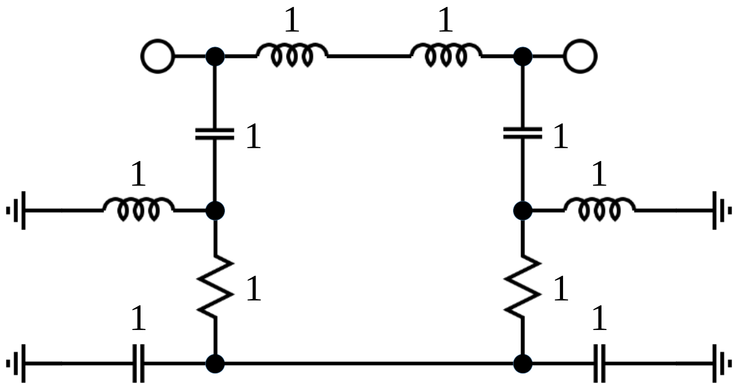
Even and Odd-Mode *Synthesis*



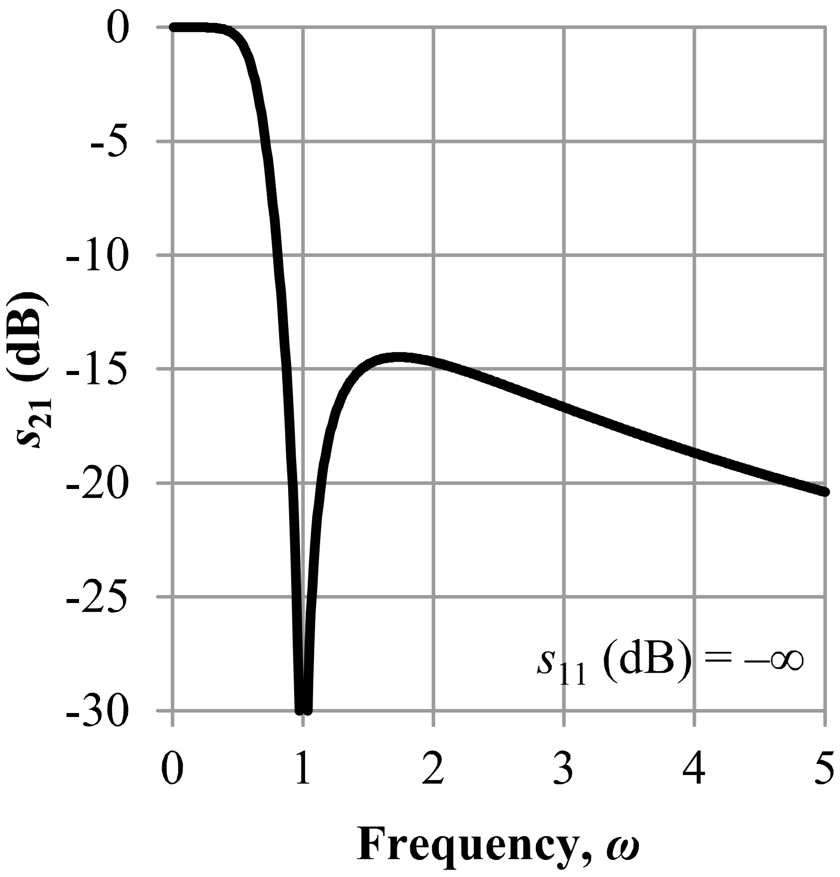
Even and Odd-Mode *Synthesis*

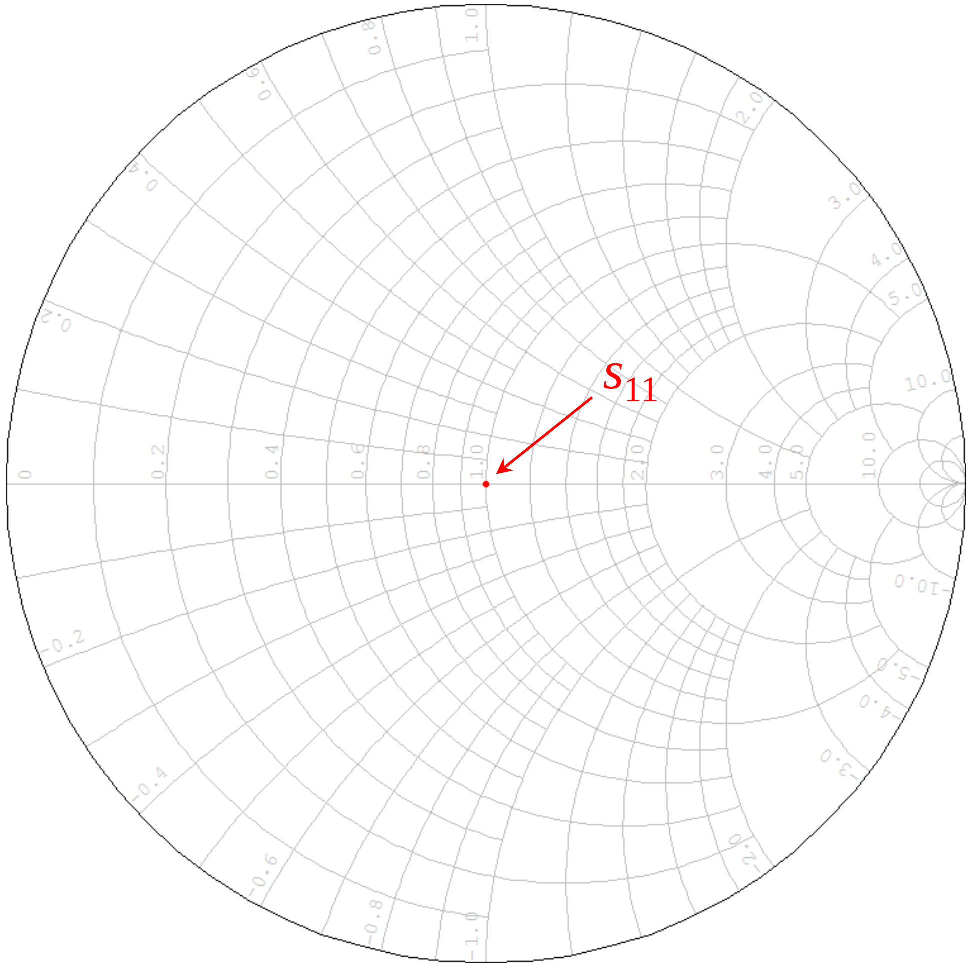


A Third-Order Reflectionless Filter

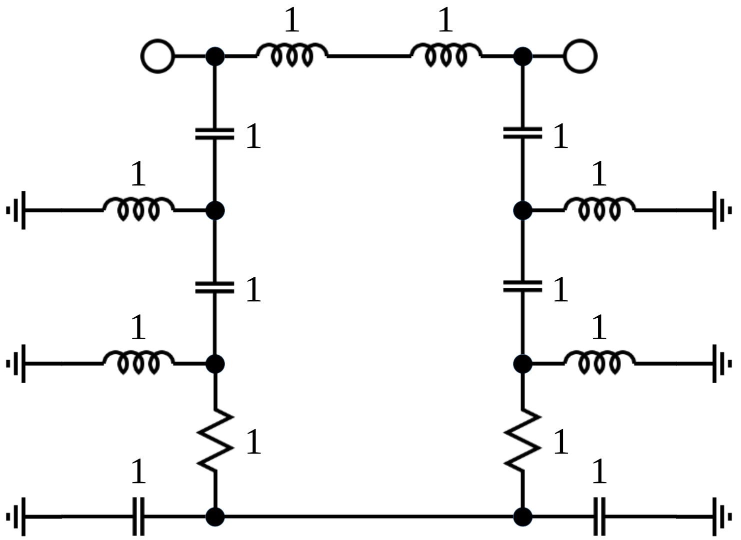


(Element values are normalized for frequency and impedance.)

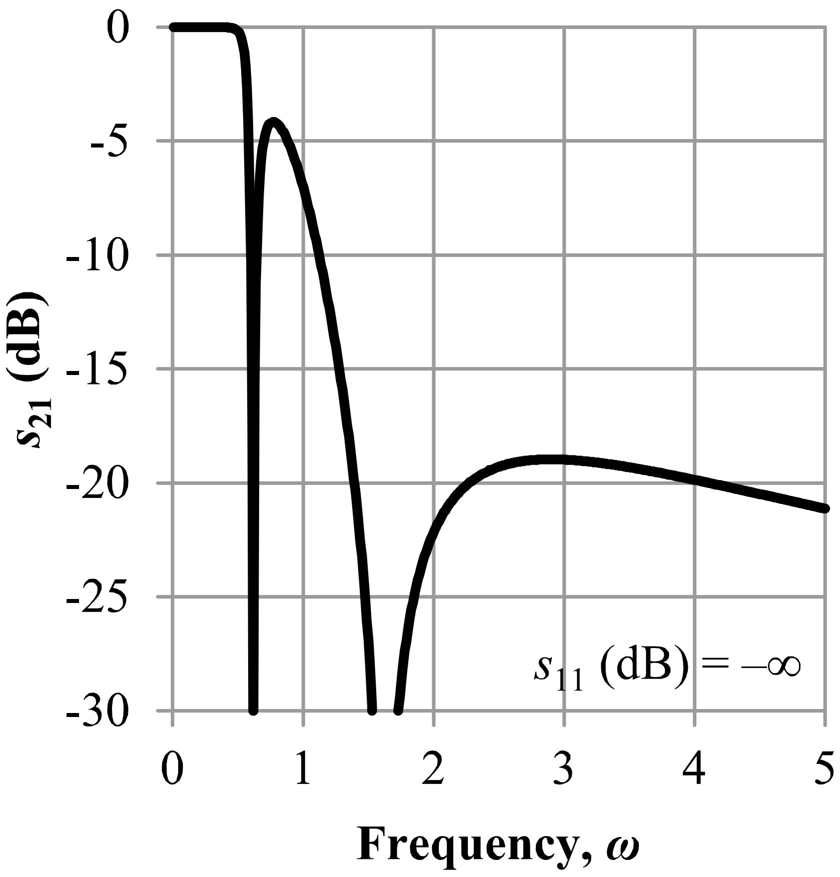


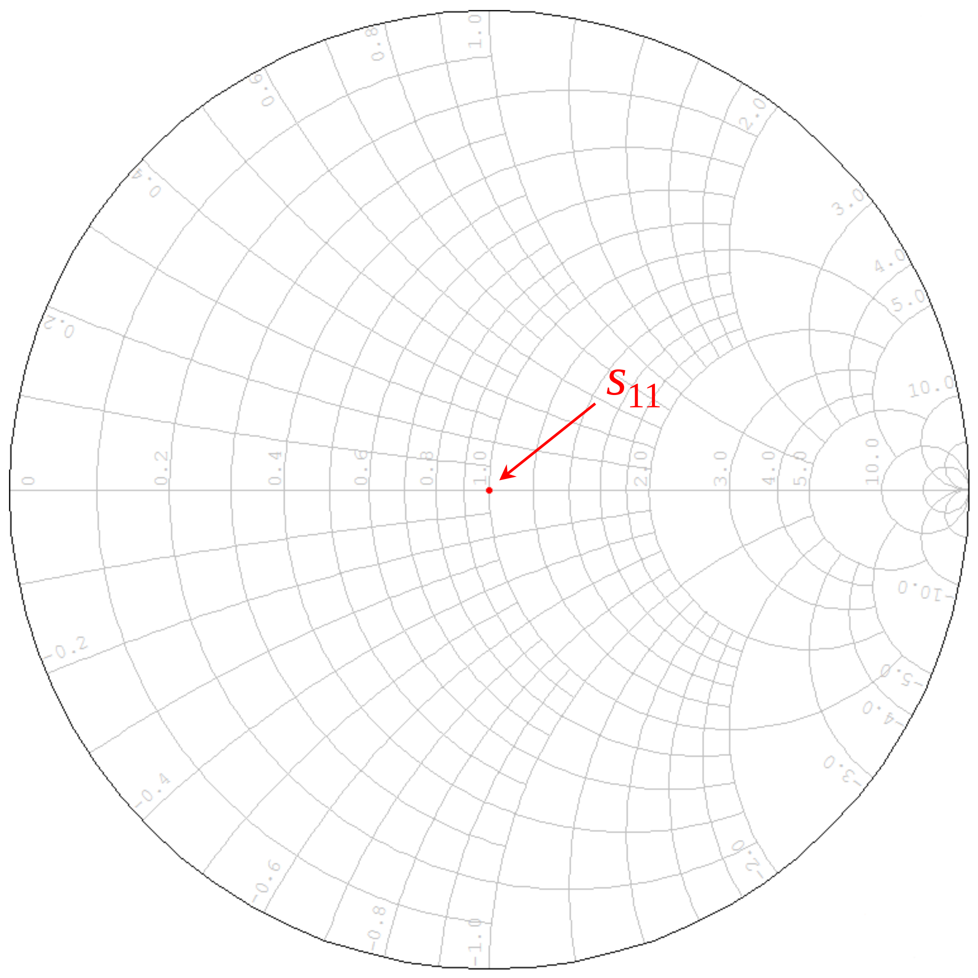


Higher-Order Responses Not So Great

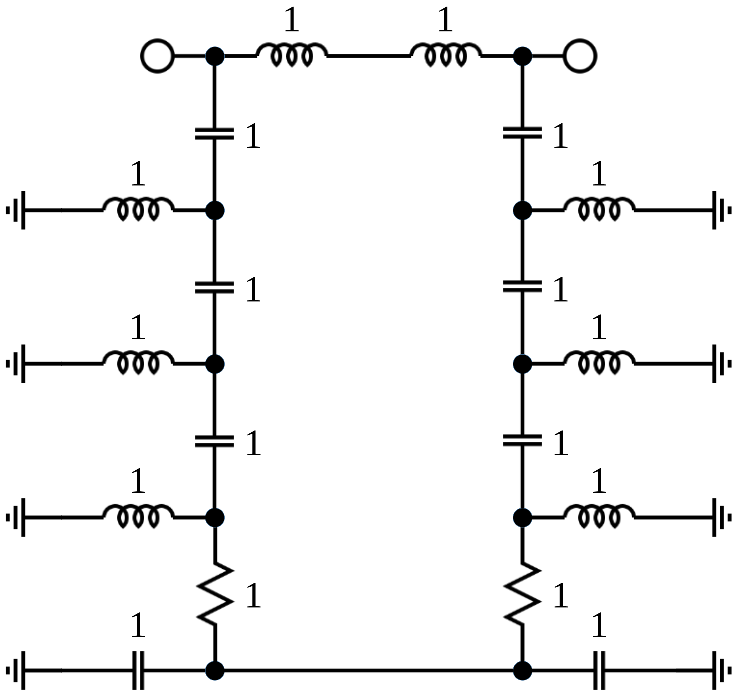


(Element values are normalized for frequency and impedance.)

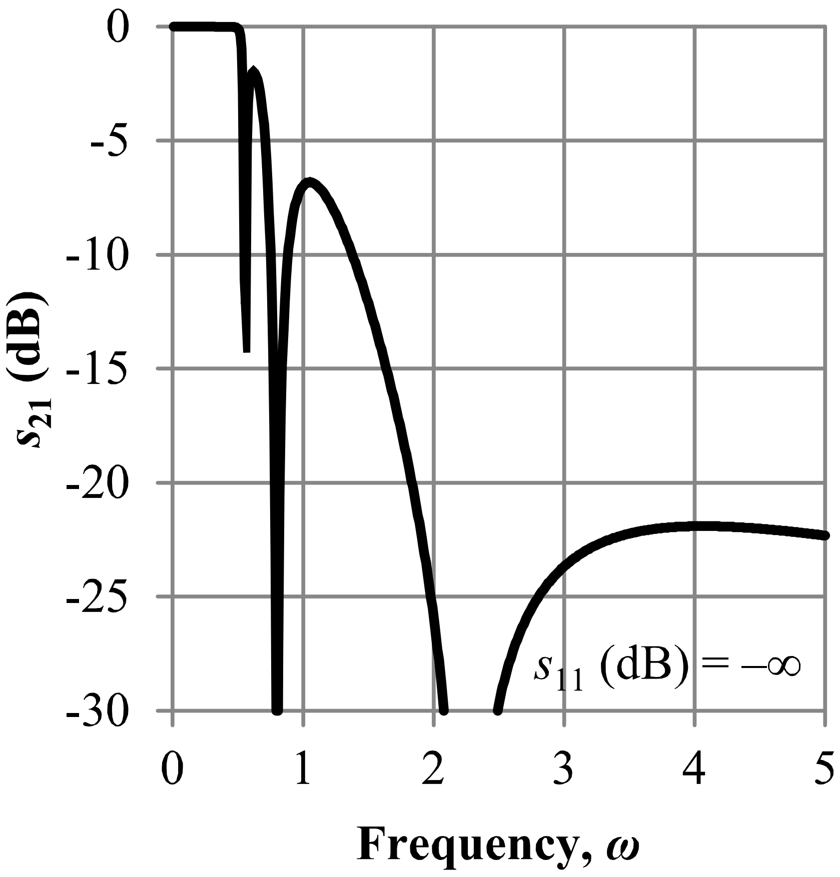


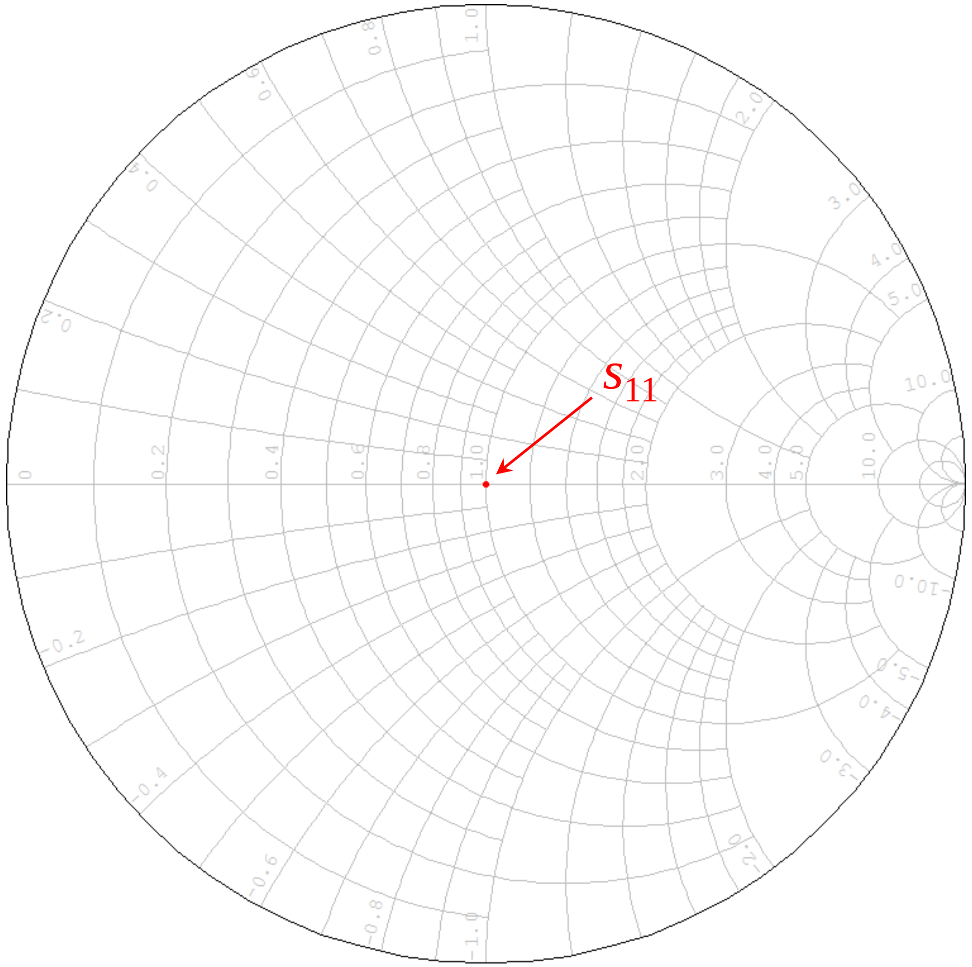


Higher-Order Responses Not So Great

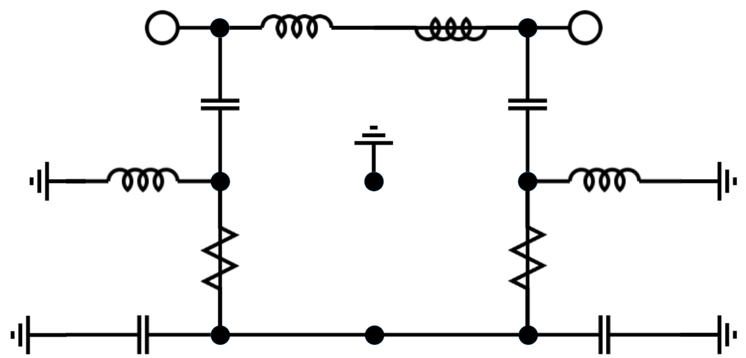


(Element values are normalized for frequency and impedance.)

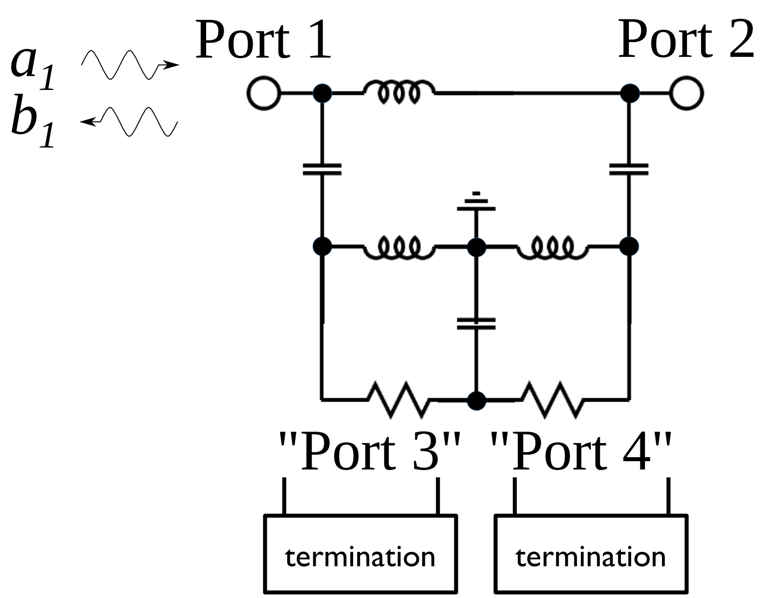


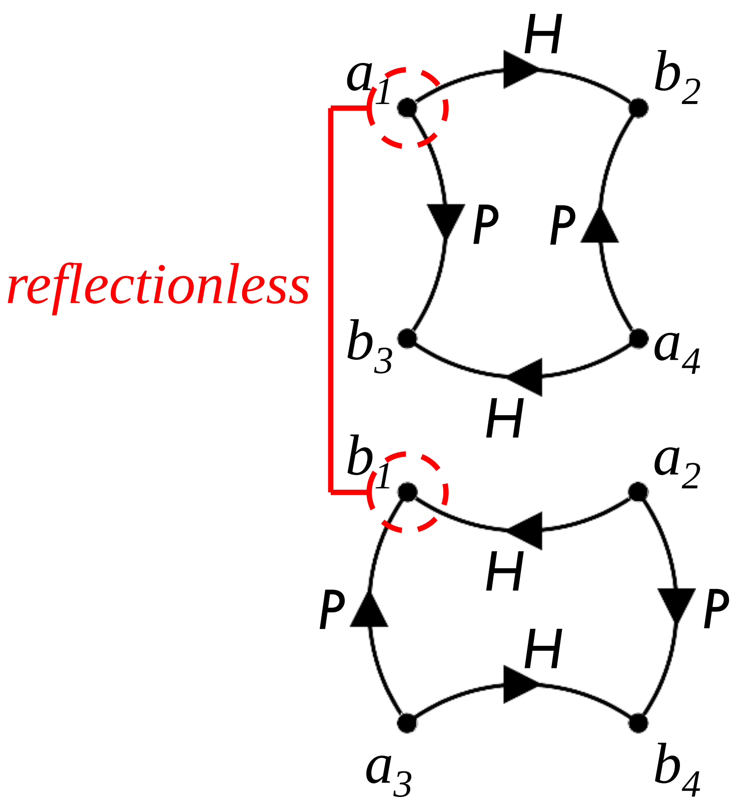


Subnetwork Expansion

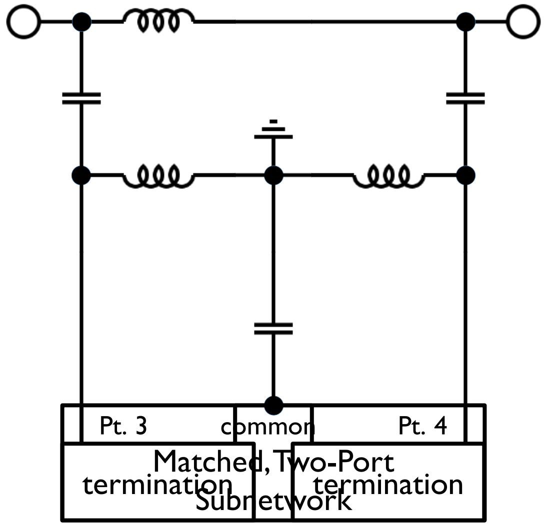


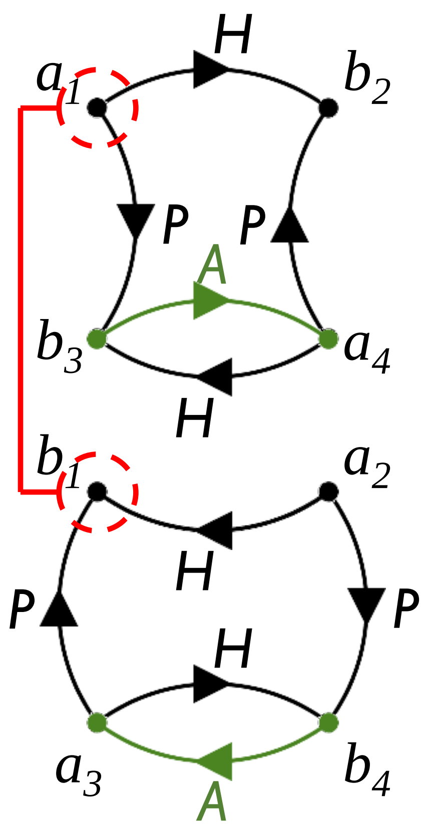
Subnetwork Expansion



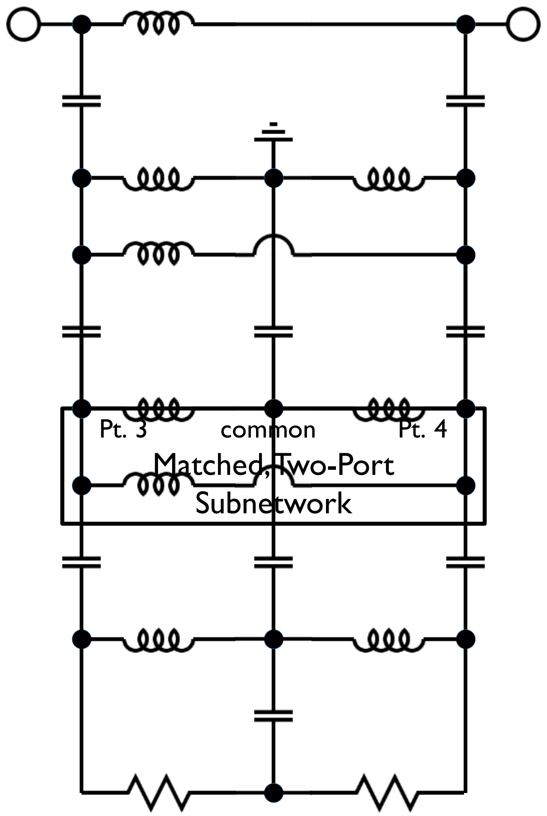


Subnetwork Expansion

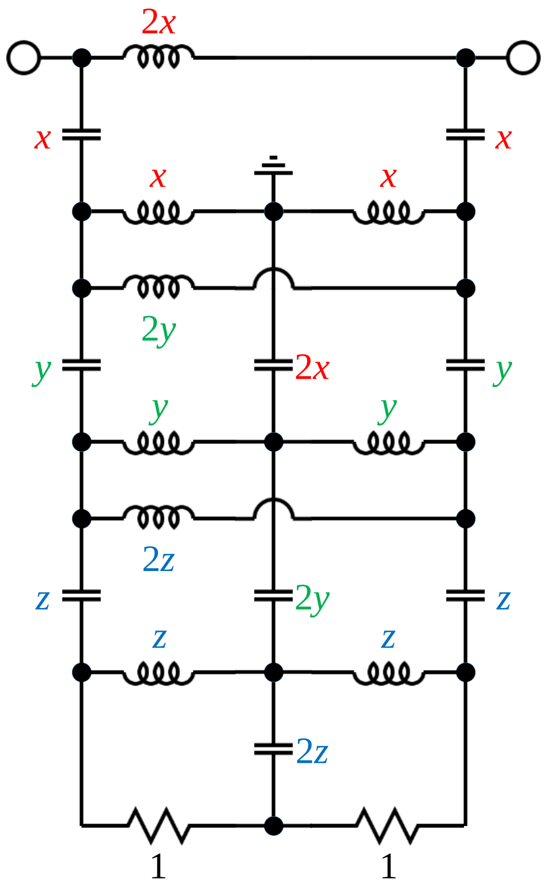




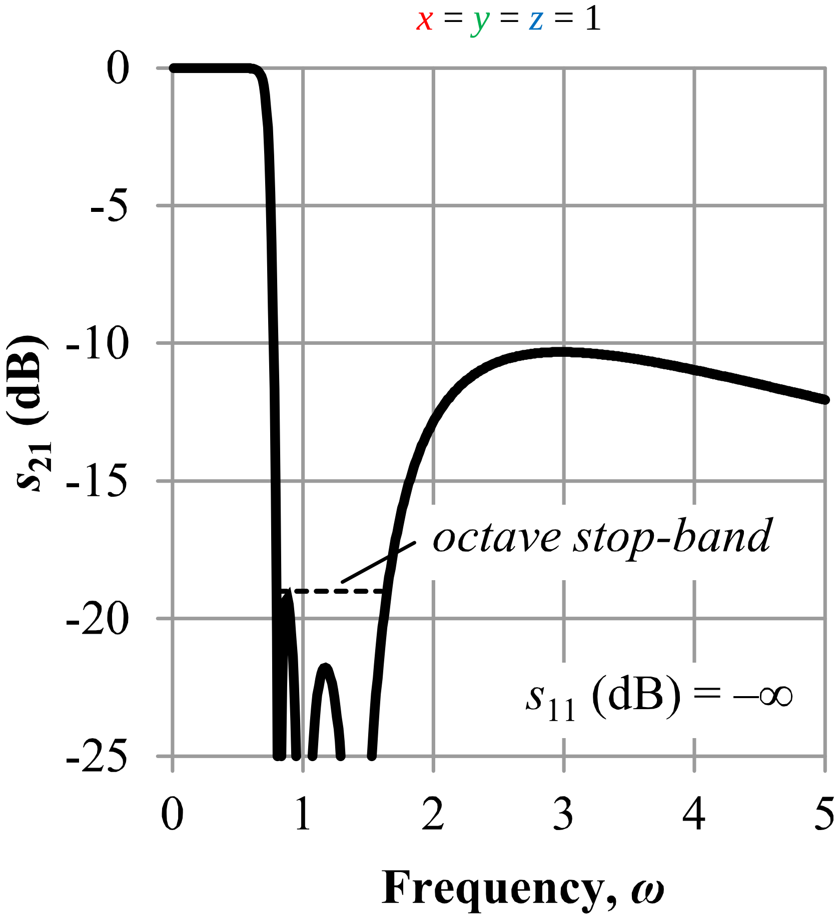
Subnetwork Expansion

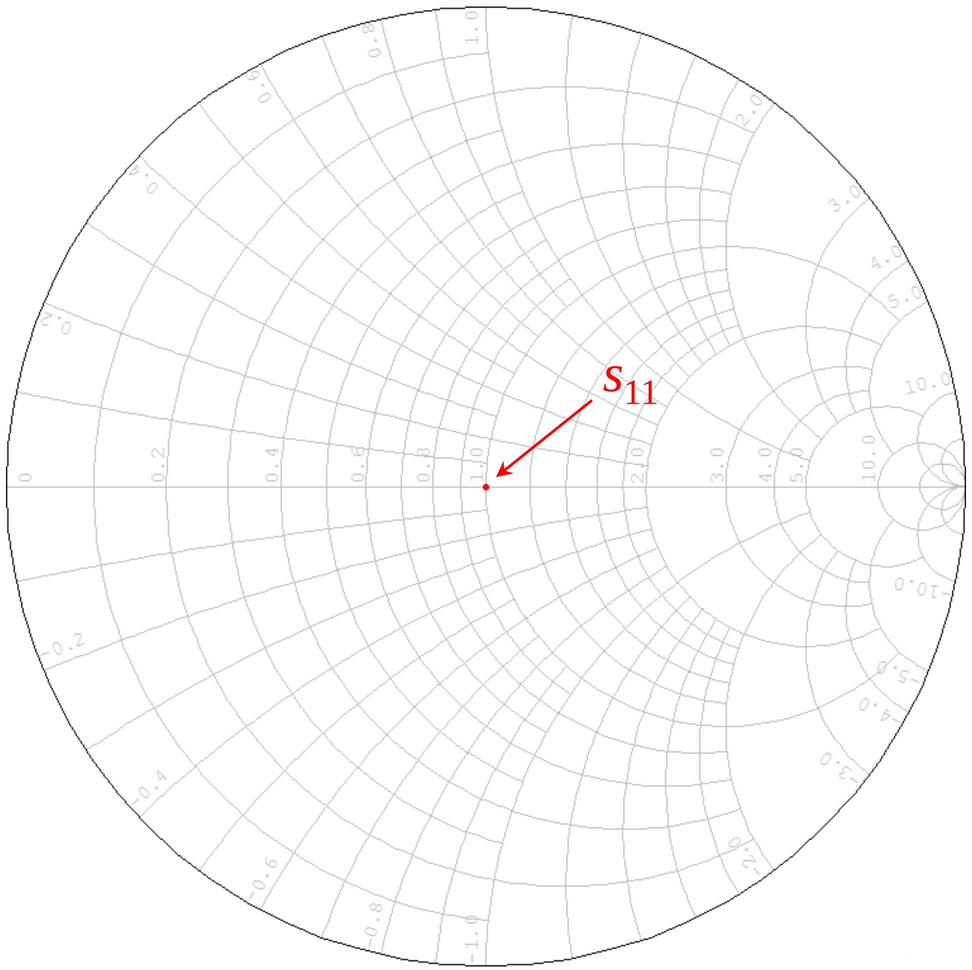


A Seventh-Order Reflectionless Filter



(Element values are normalized
for frequency and impedance.)

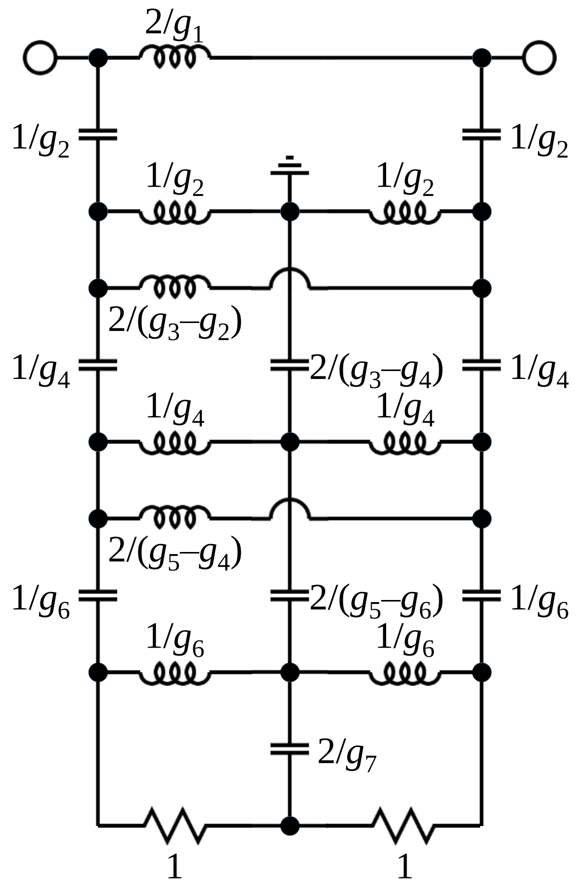




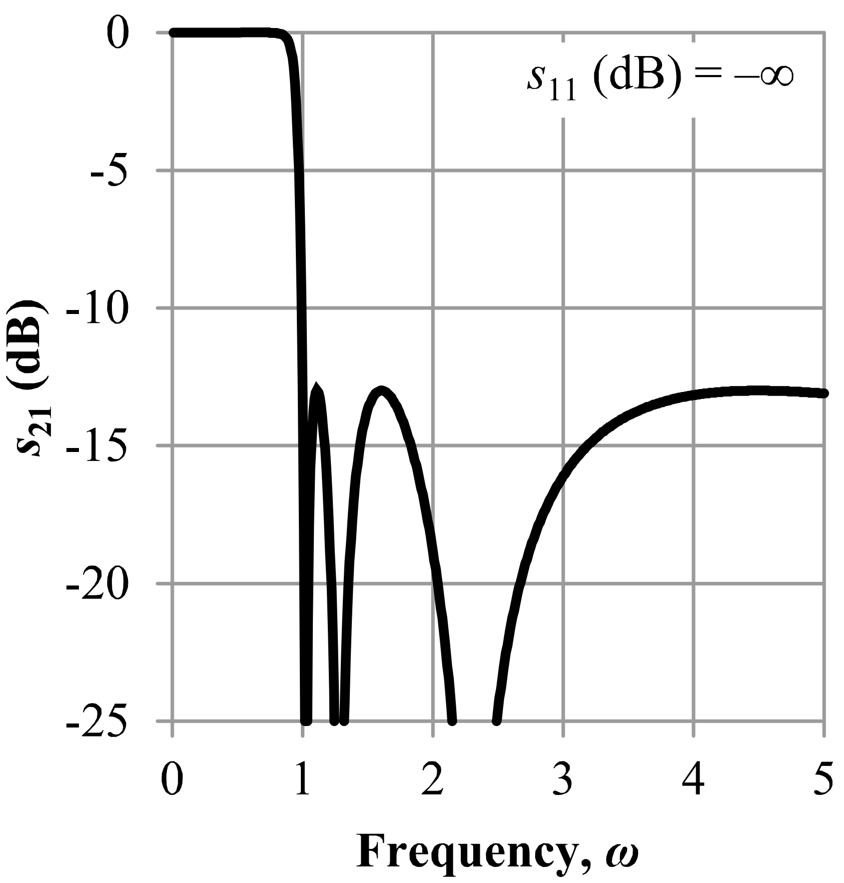
Element Value Generalization

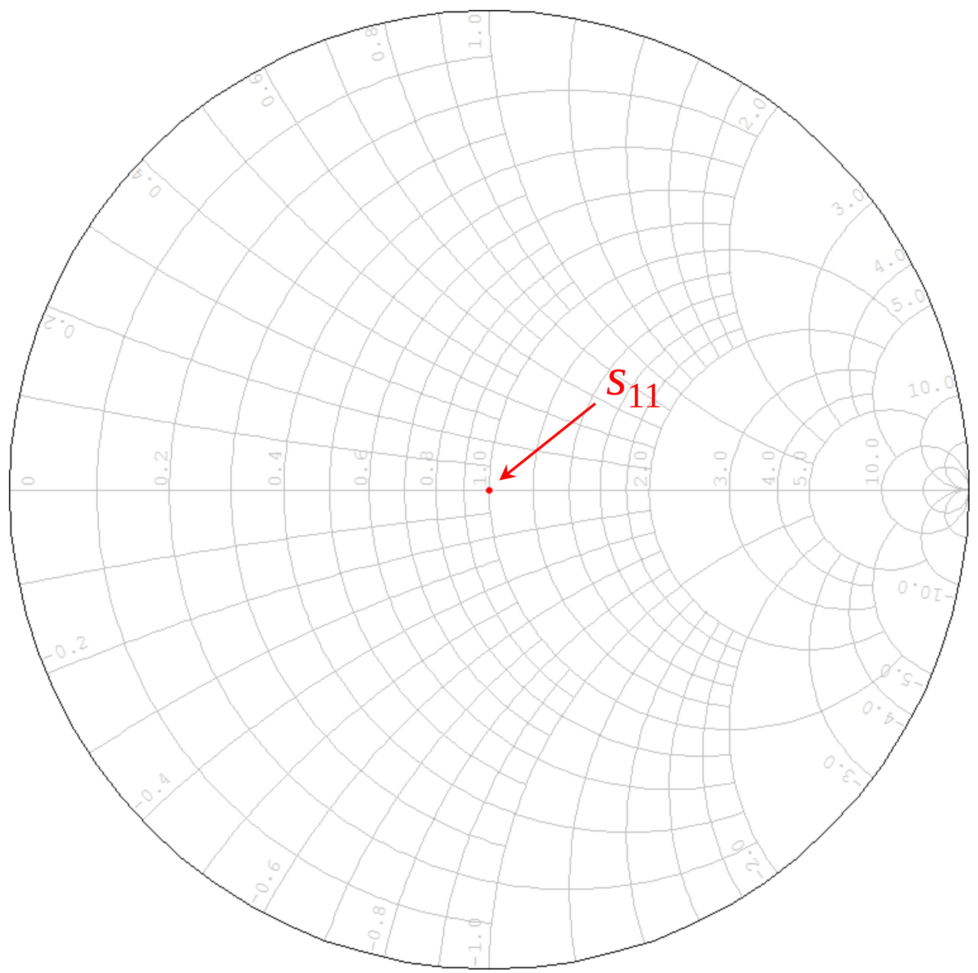
$g_1 = g_2$

$g_6 = g_7$

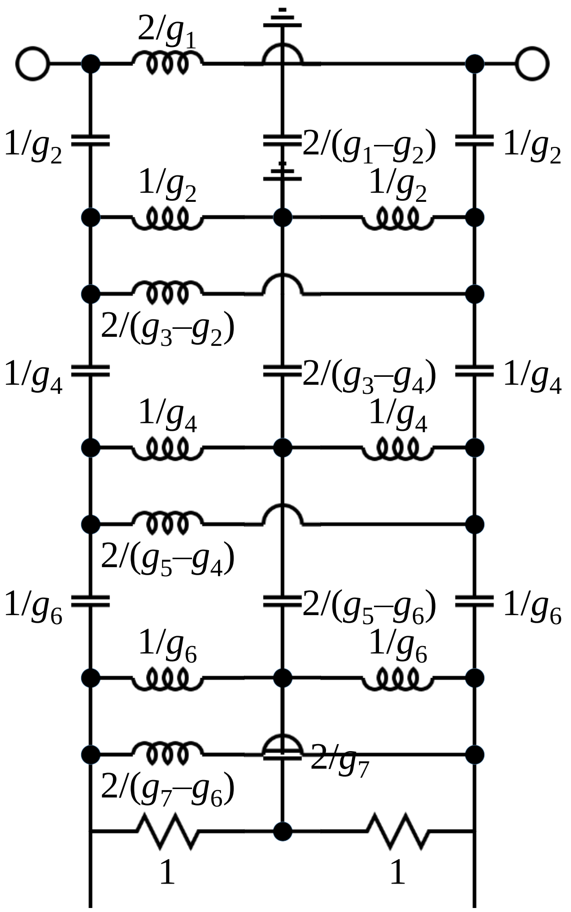


(Element values are normalized for frequency and impedance.)

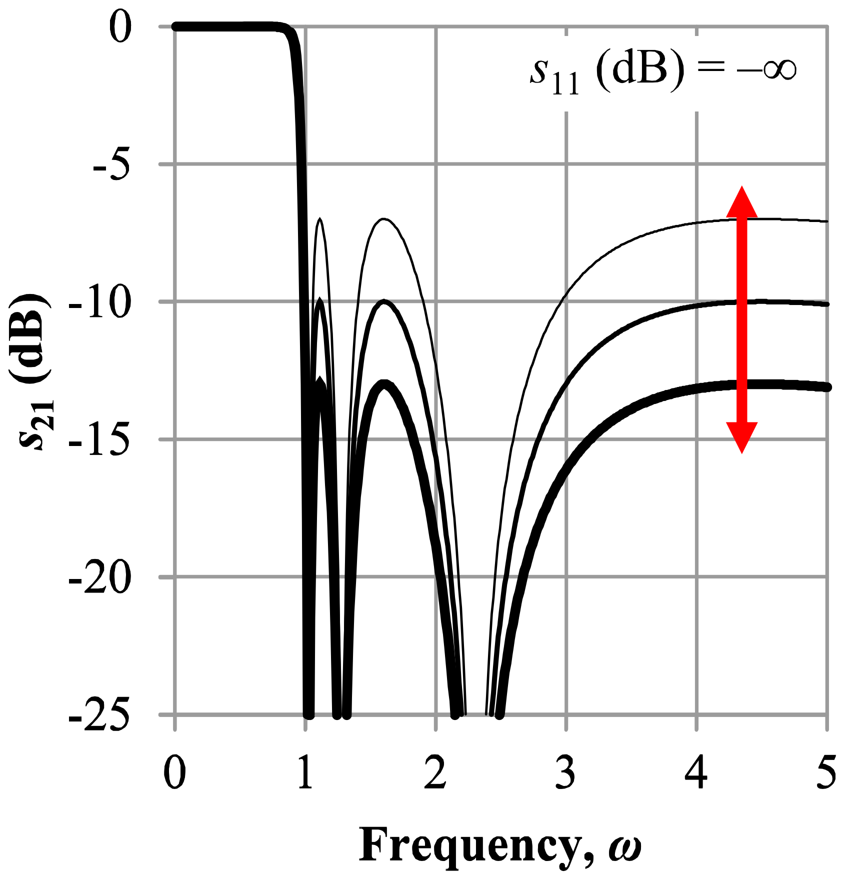


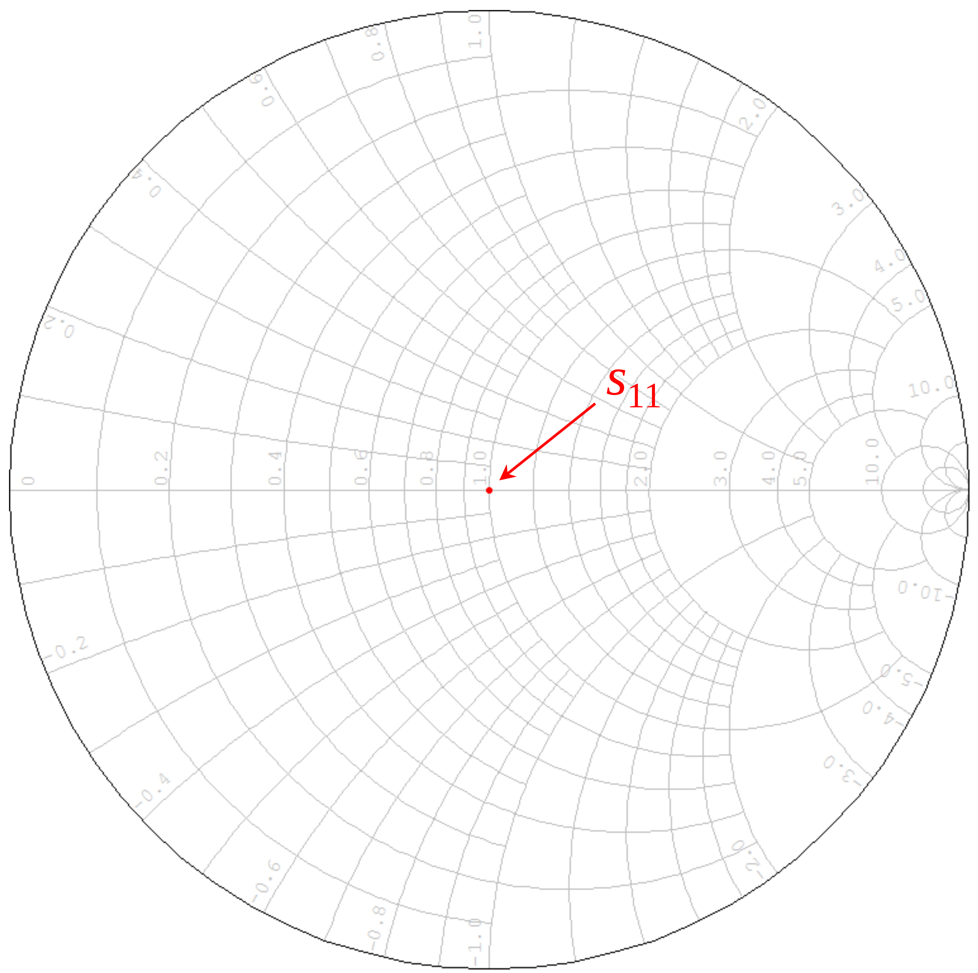


Topological Generalization

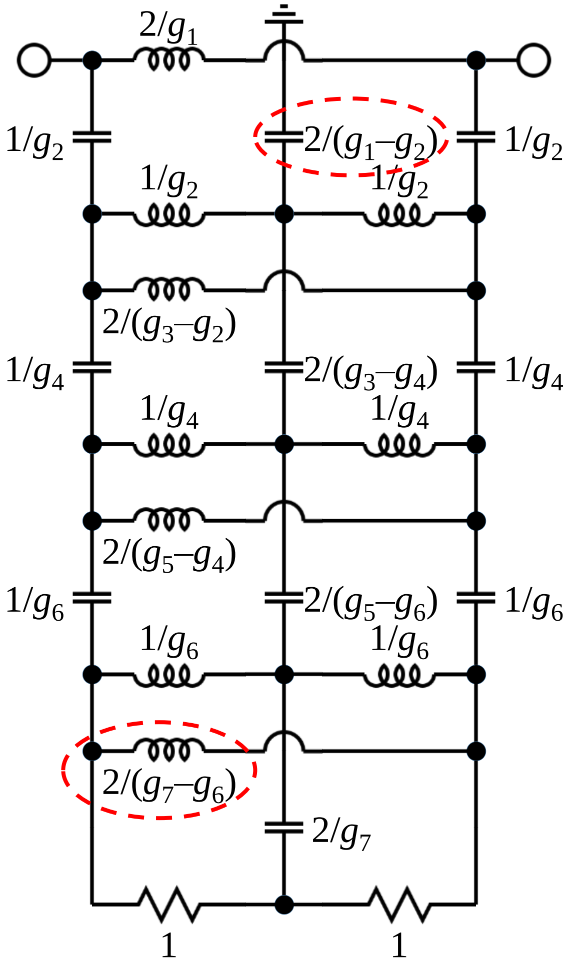


(Element values are normalized for frequency and impedance.)

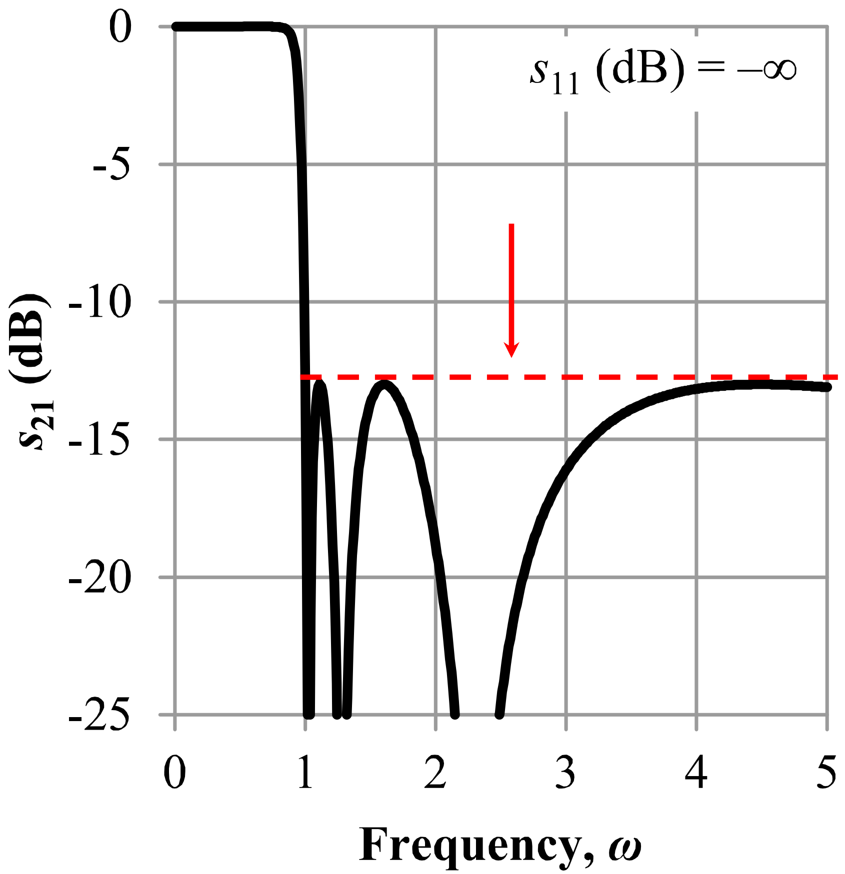


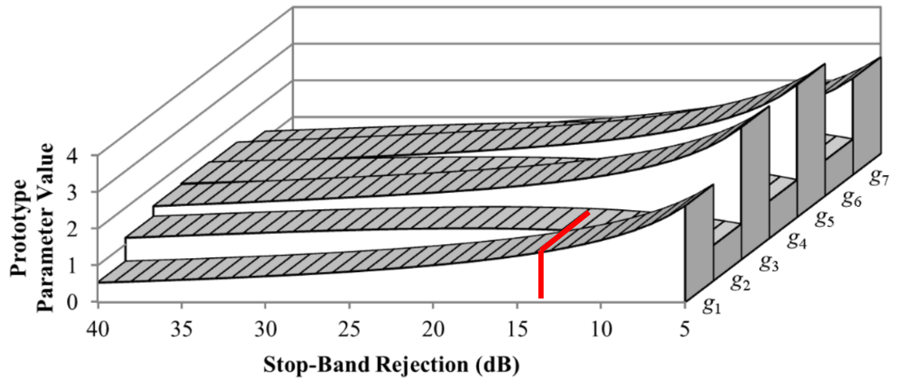


Limiting Ripple Factor

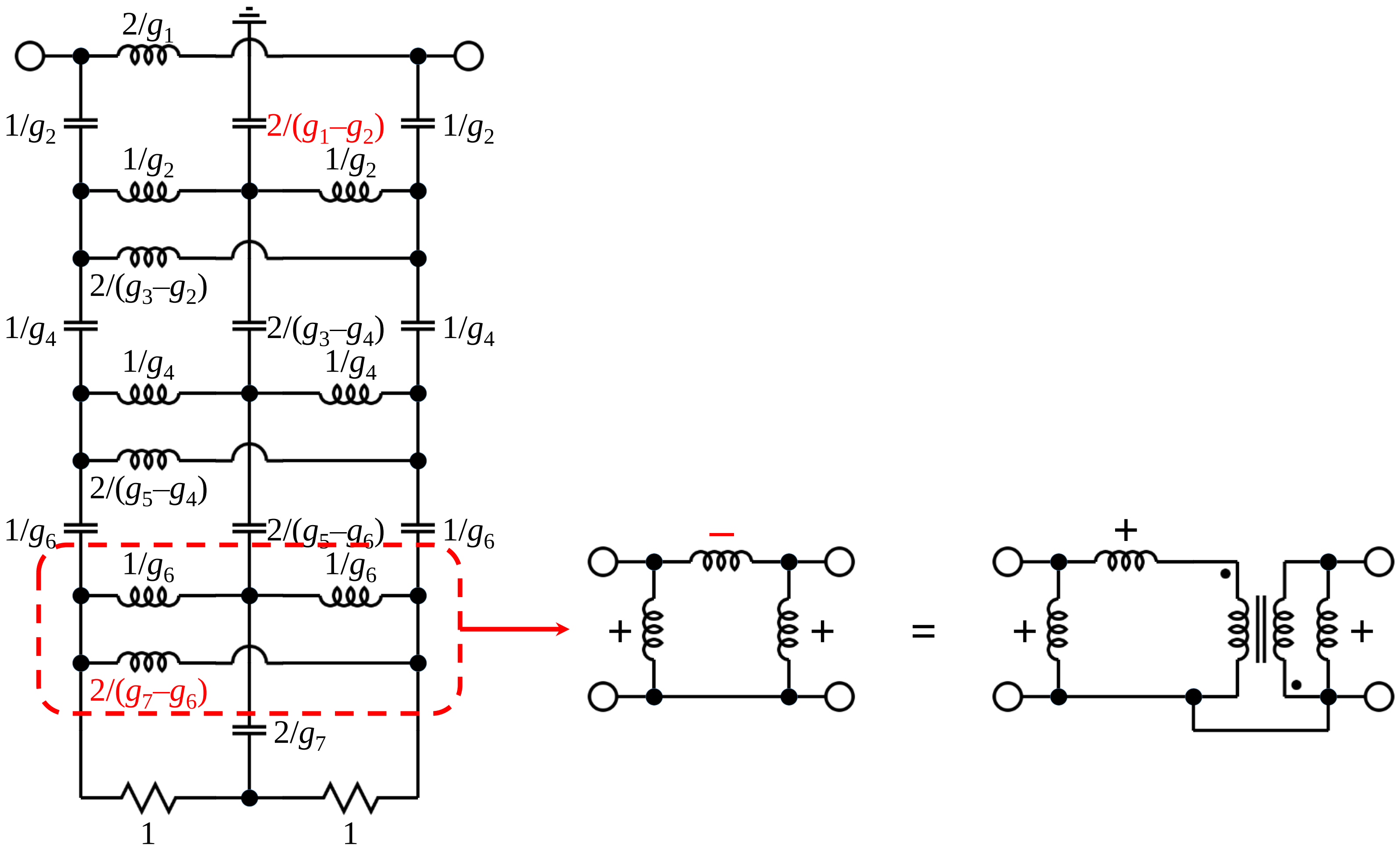


(Element values are normalized for frequency and impedance.)



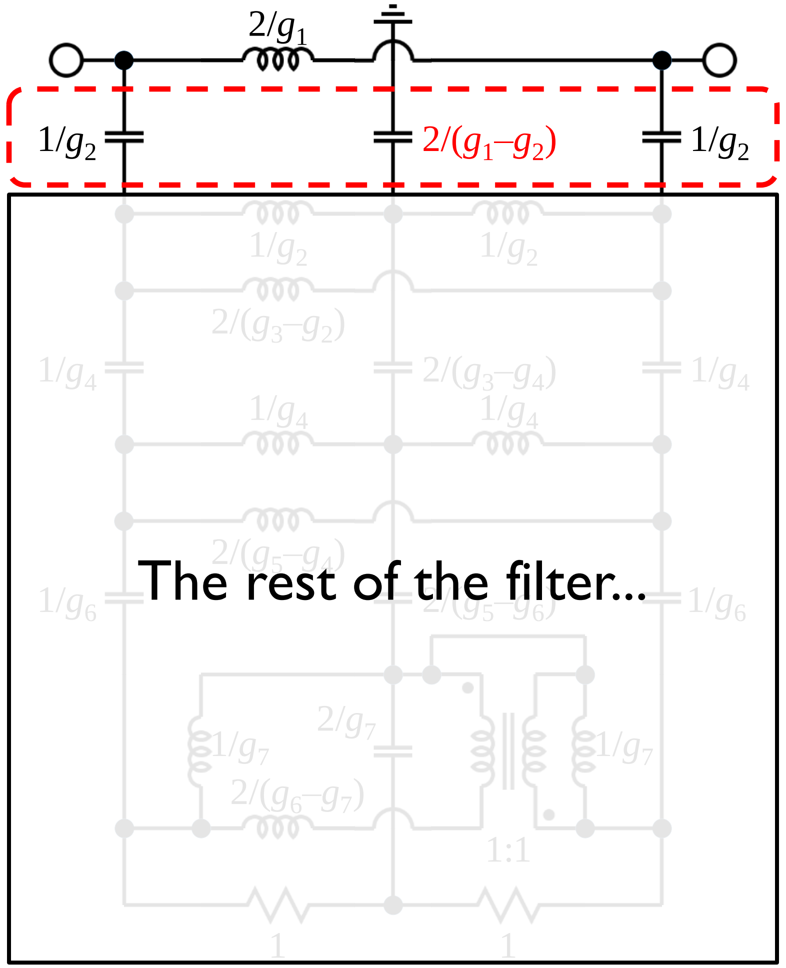


Negative Element Mitigation



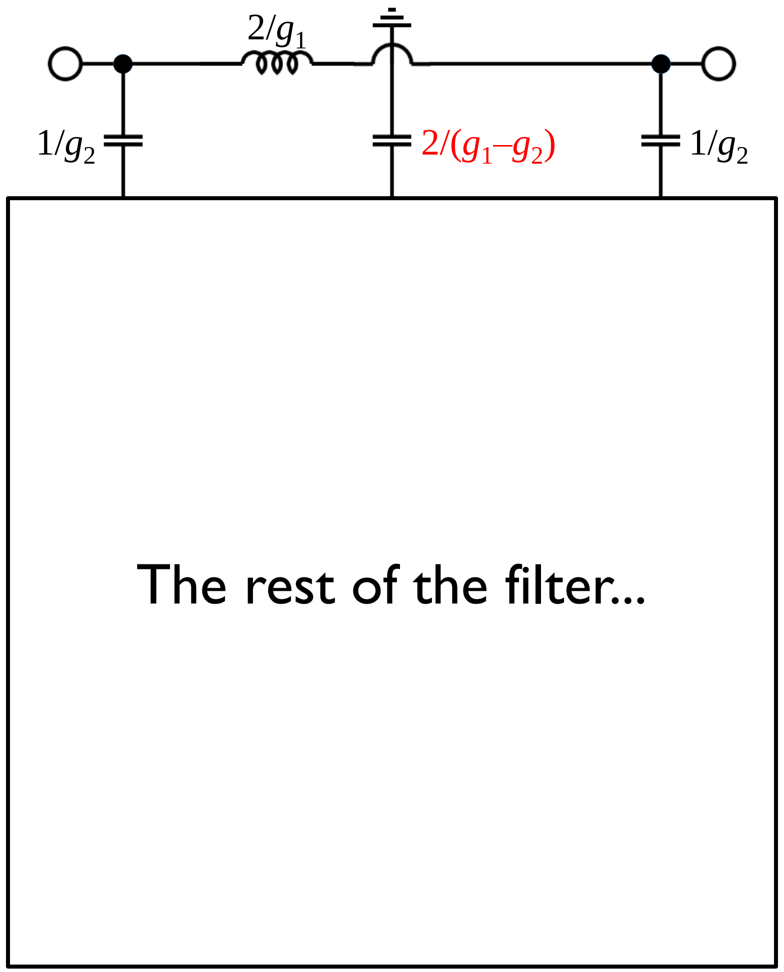
(Element values are normalized for frequency and impedance.)

Negative Element Mitigation



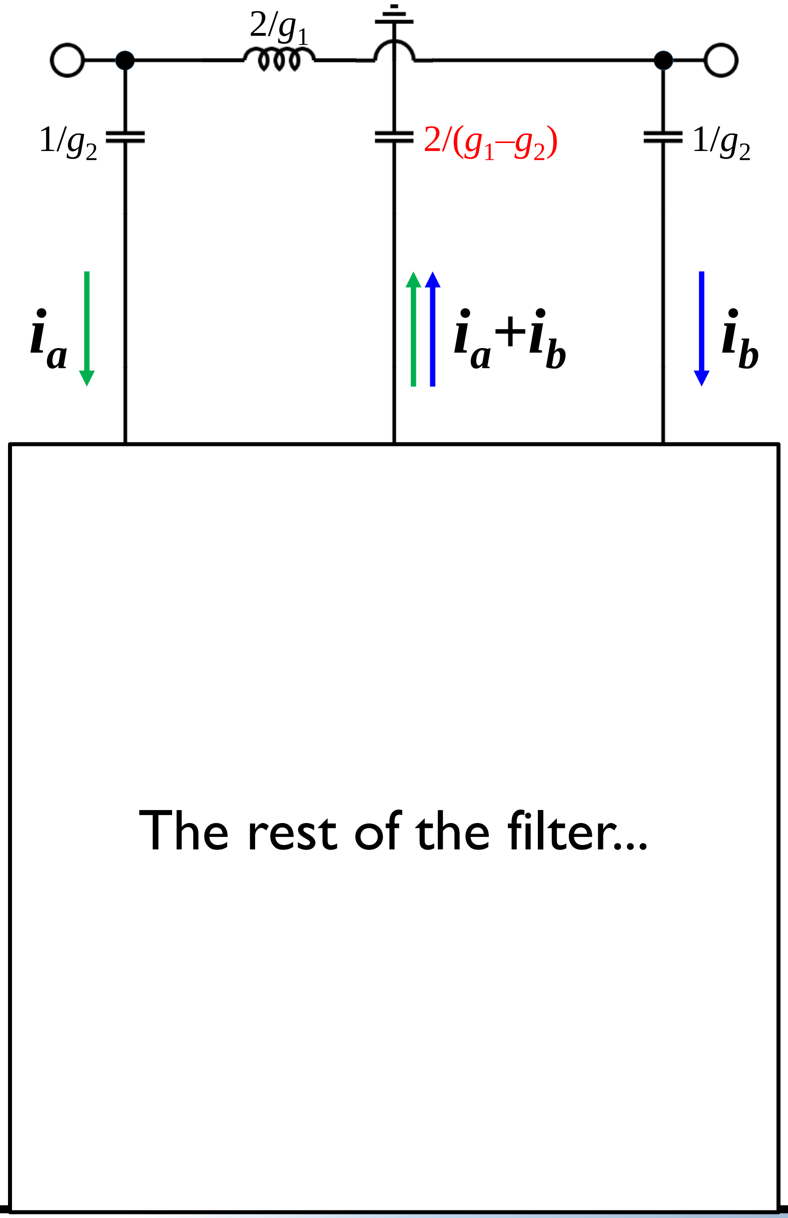
(Element values are normalized for frequency and impedance.)

Negative Element Mitigation

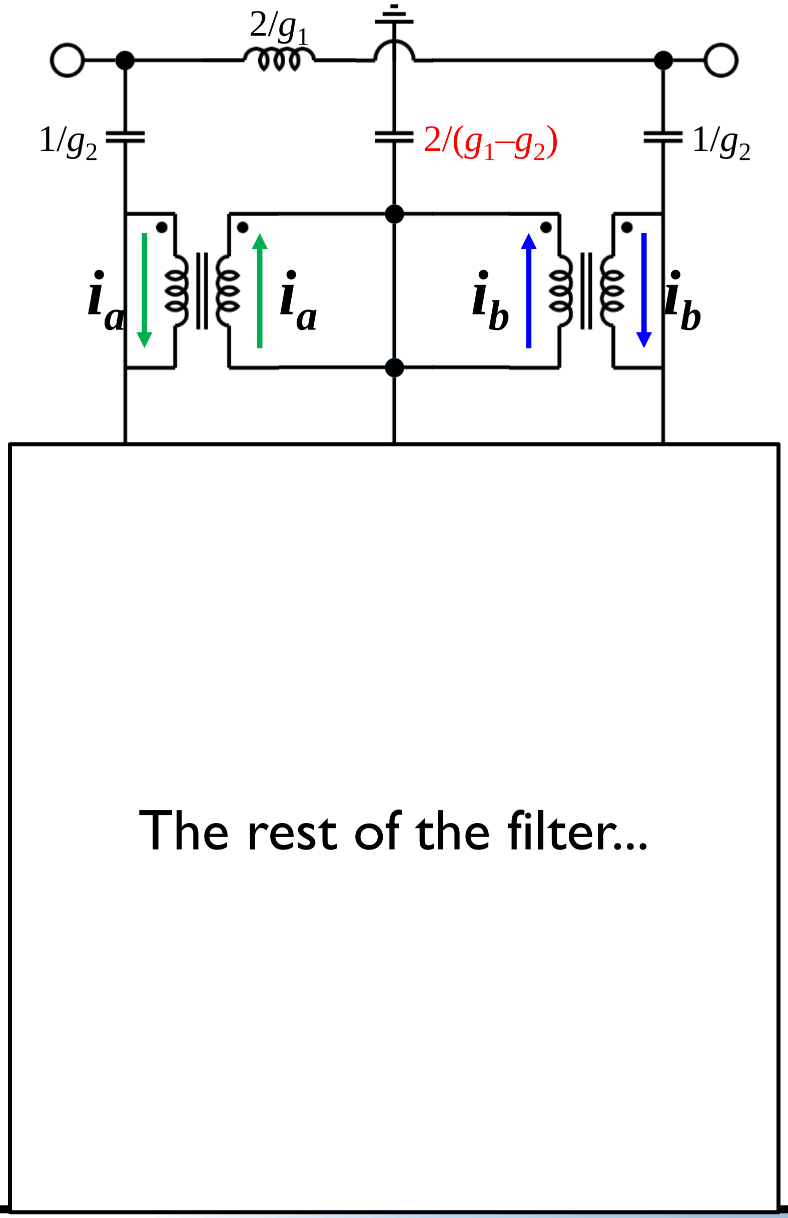


(Element values are normalized for frequency and impedance.)

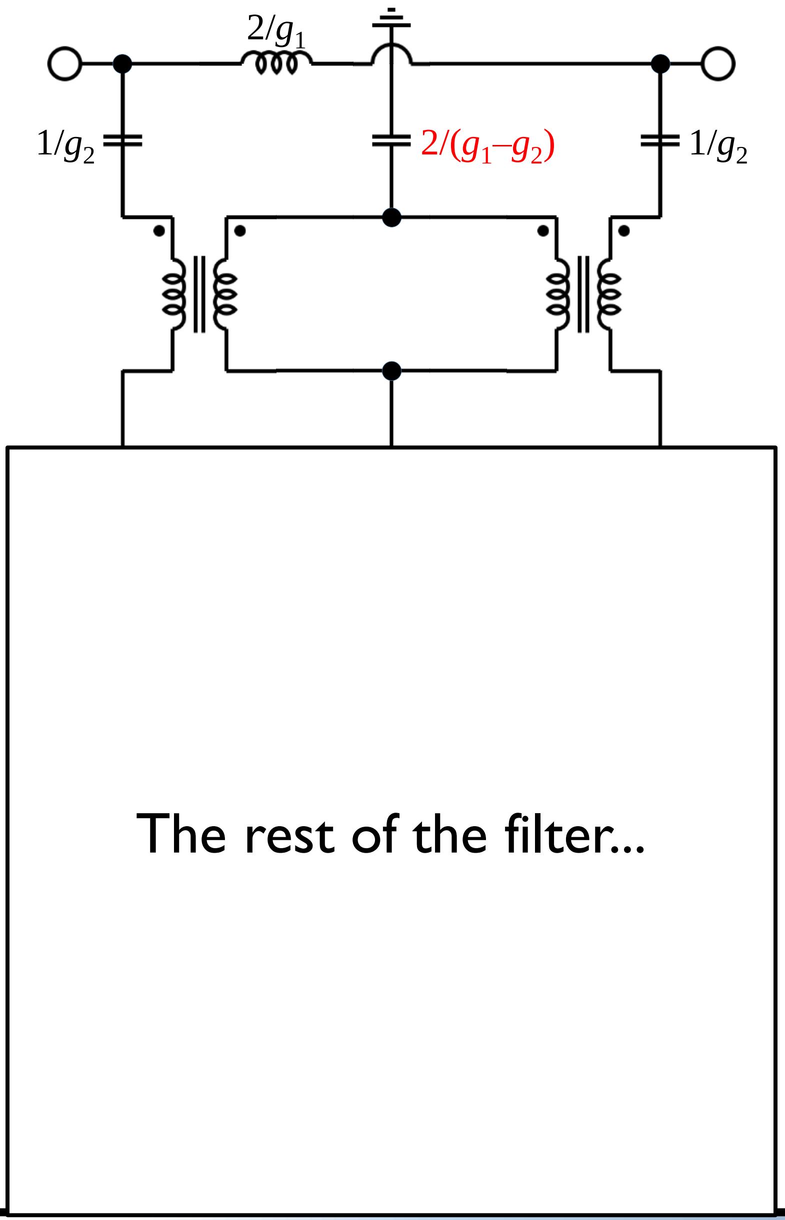
Negative Element Mitigation

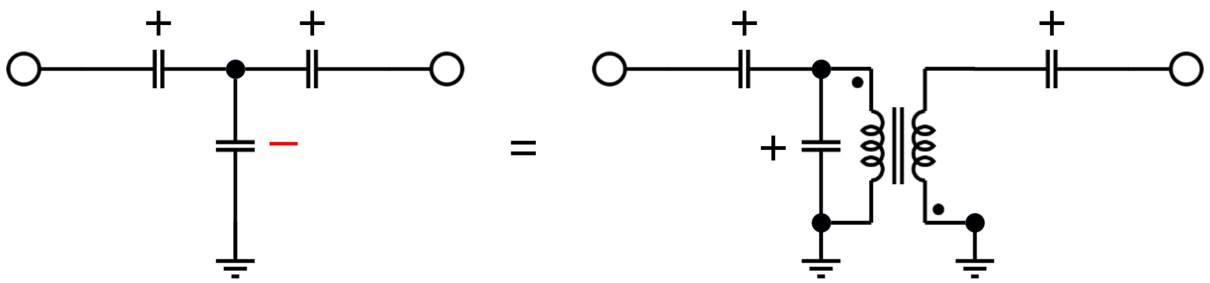


Negative Element Mitigation

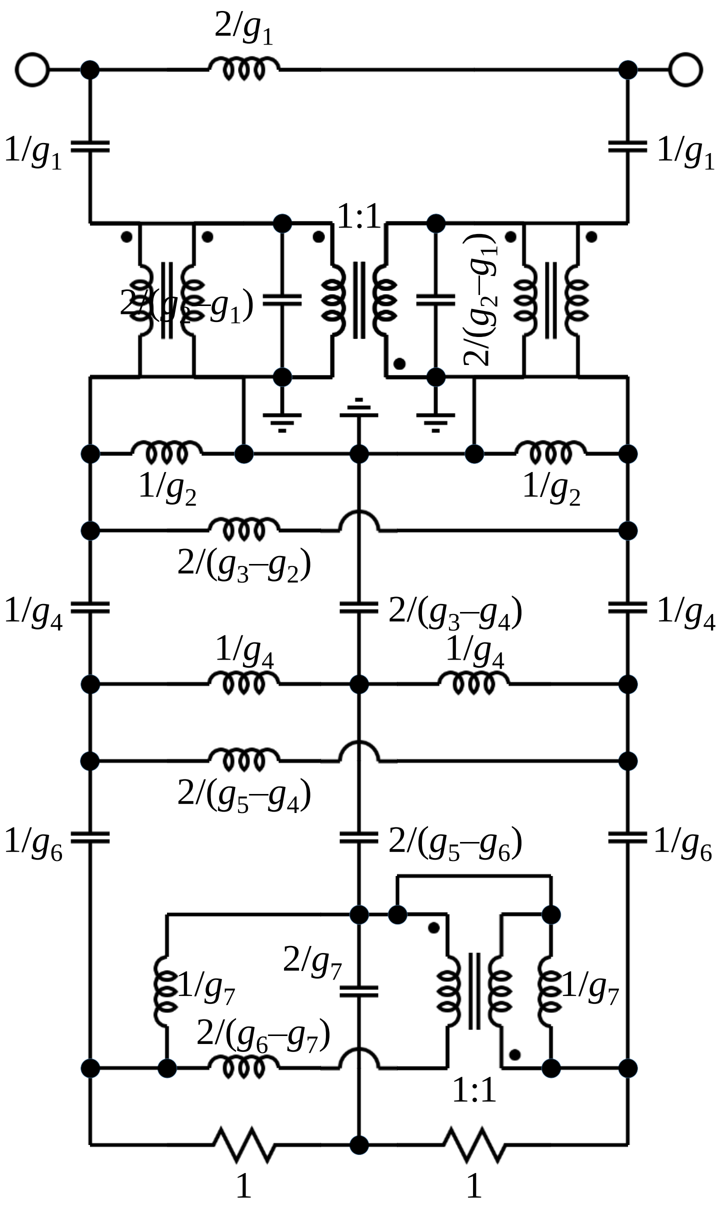


Negative Element Mitigation



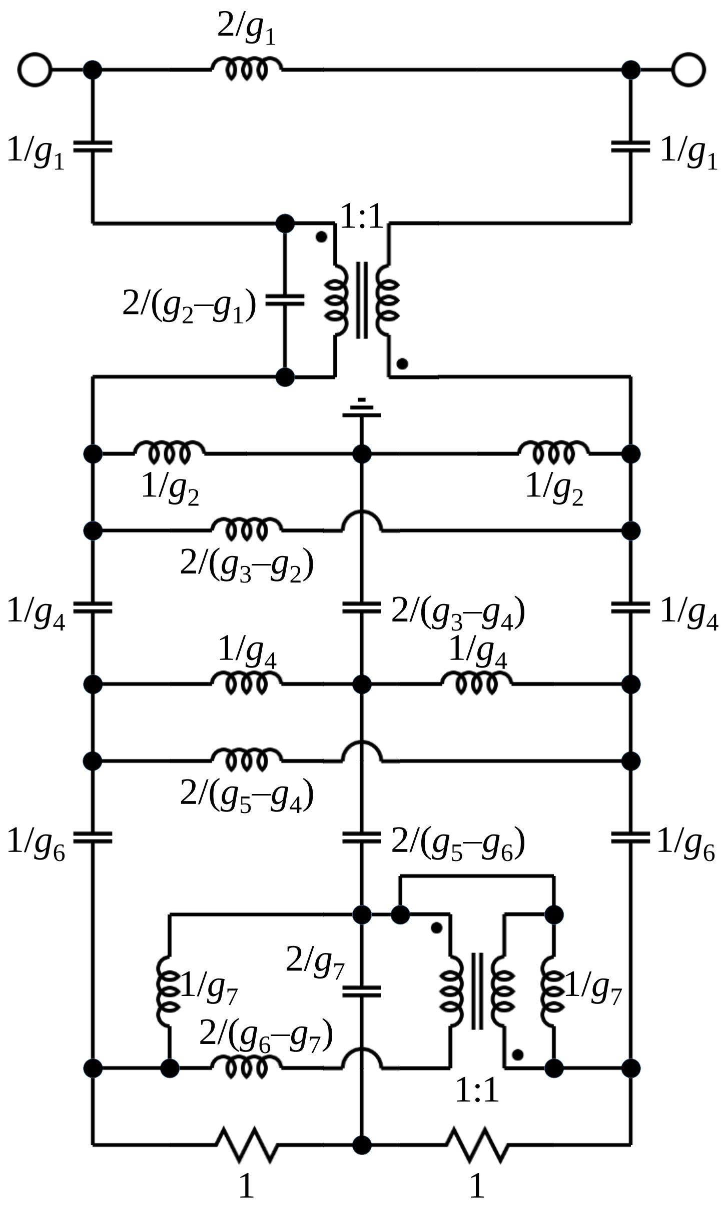


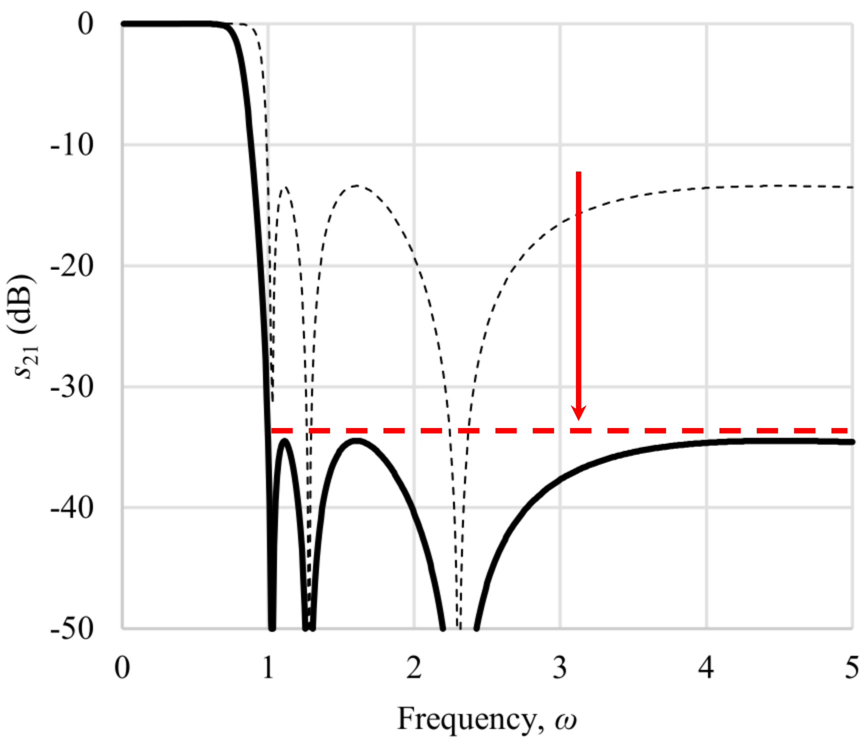
Negative Element Mitigation

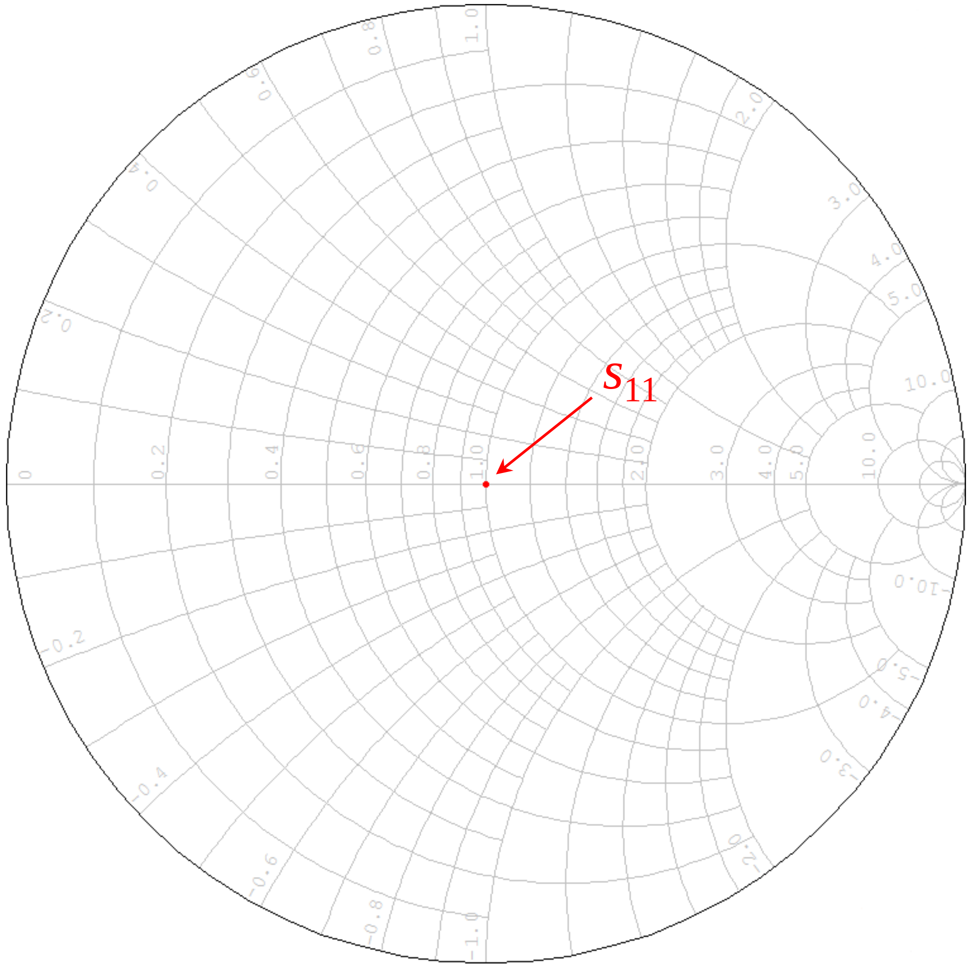


A little redundant...

No More Negative Elements!

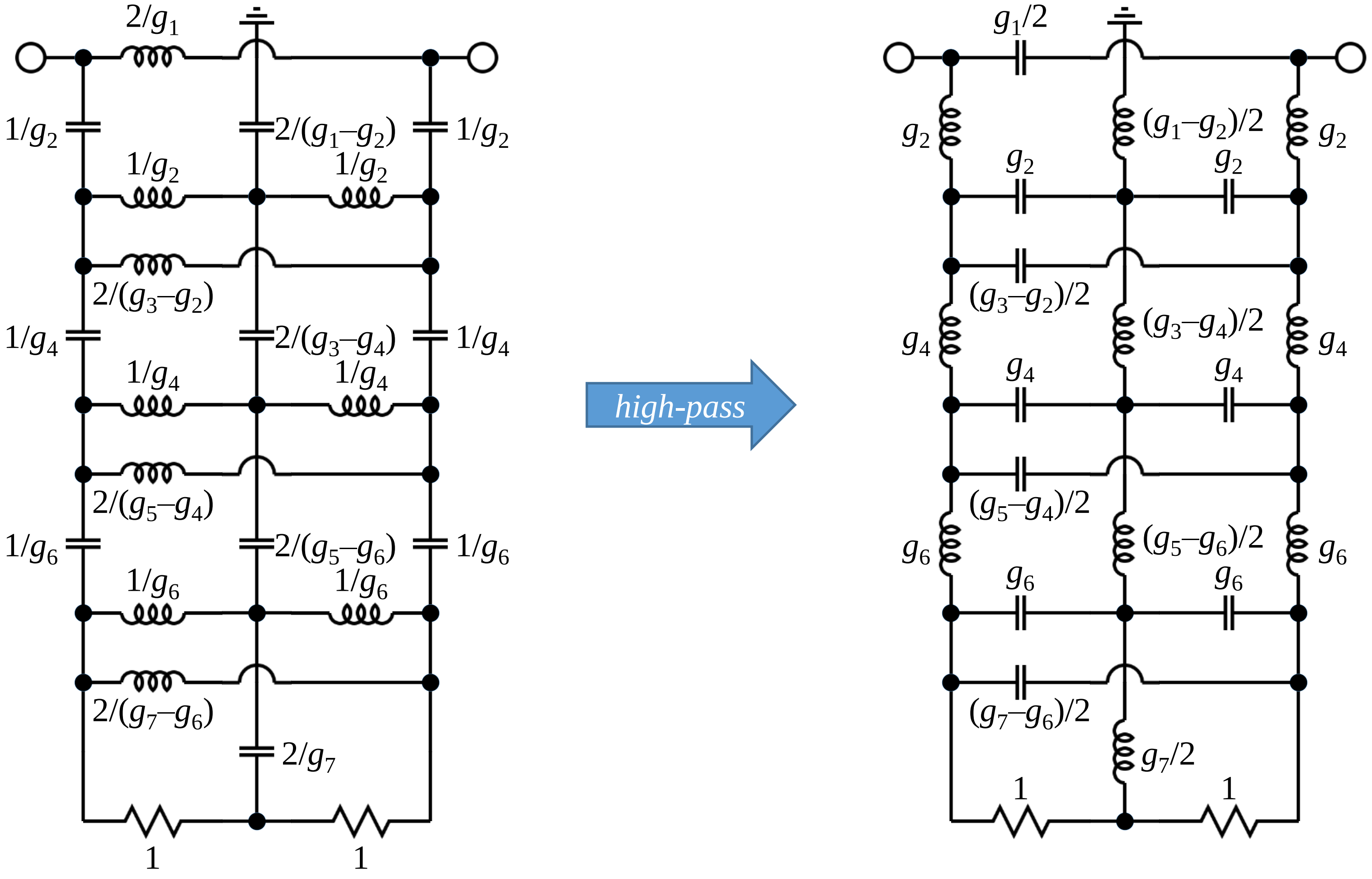




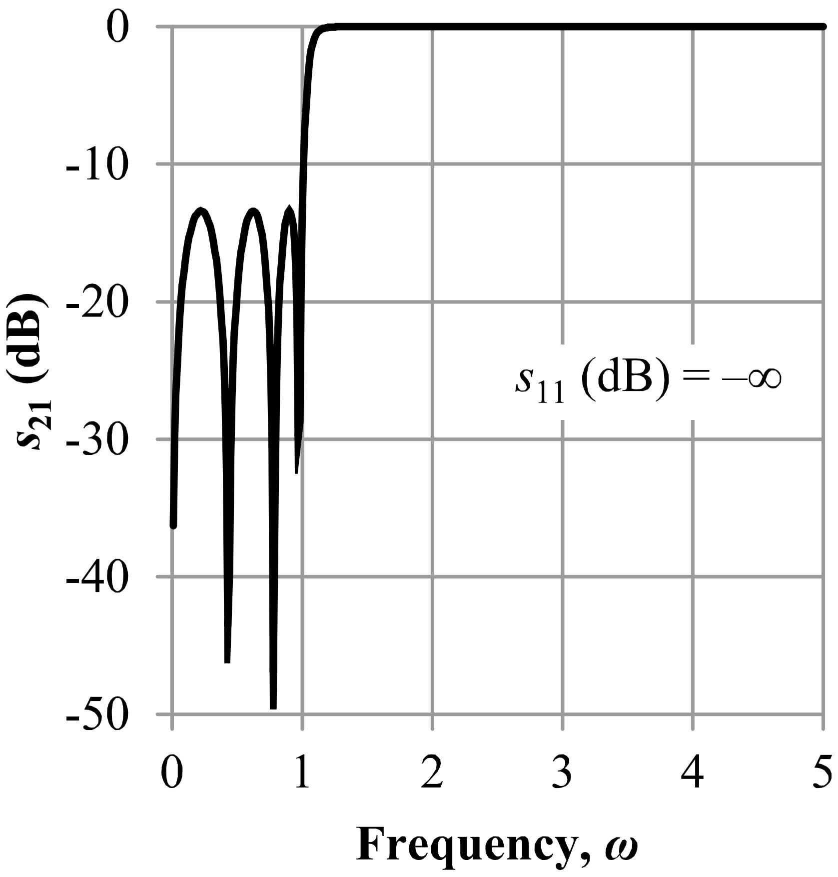


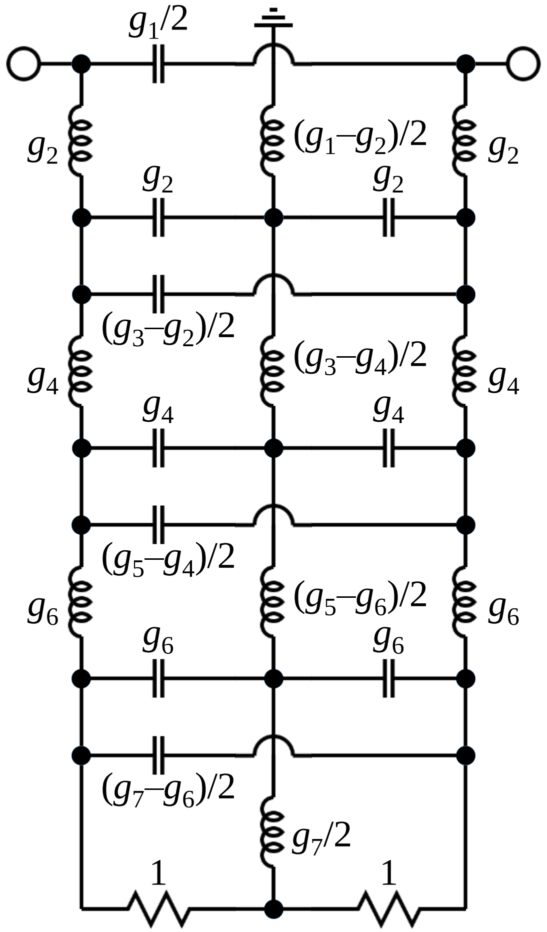
All-Pole Topologies

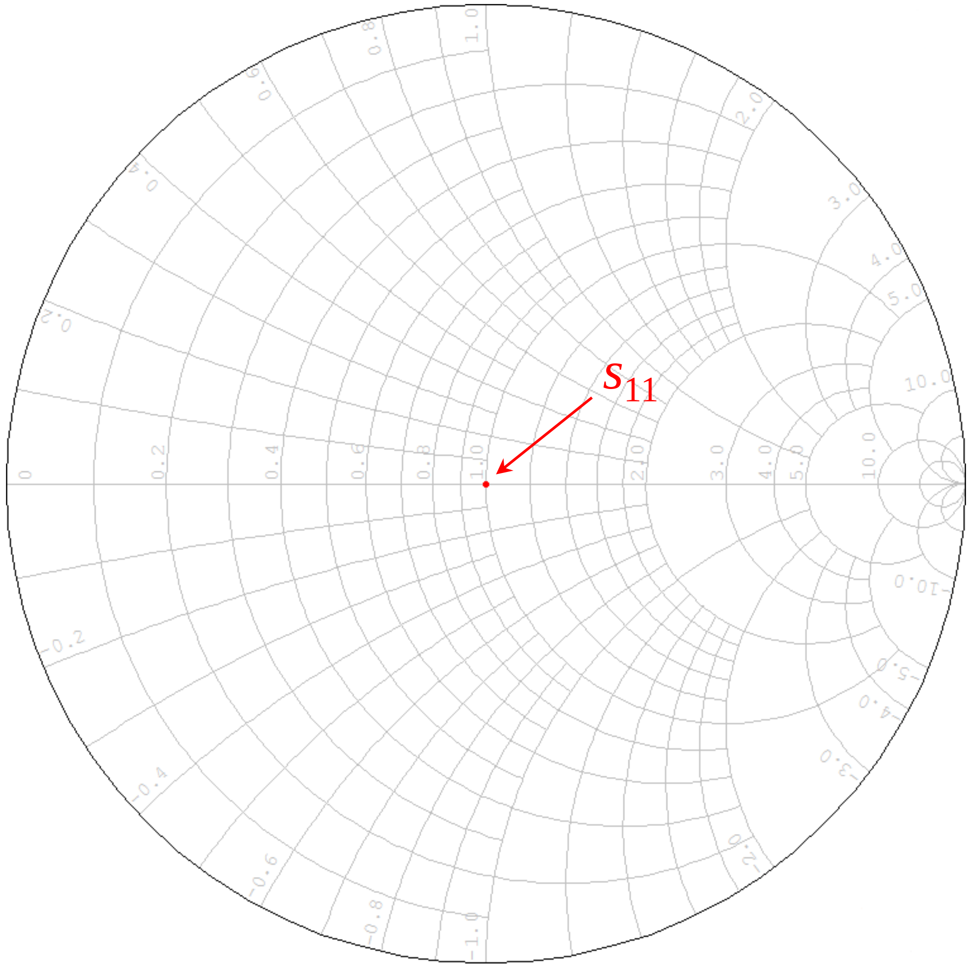
Type-II to Type-I Transformation



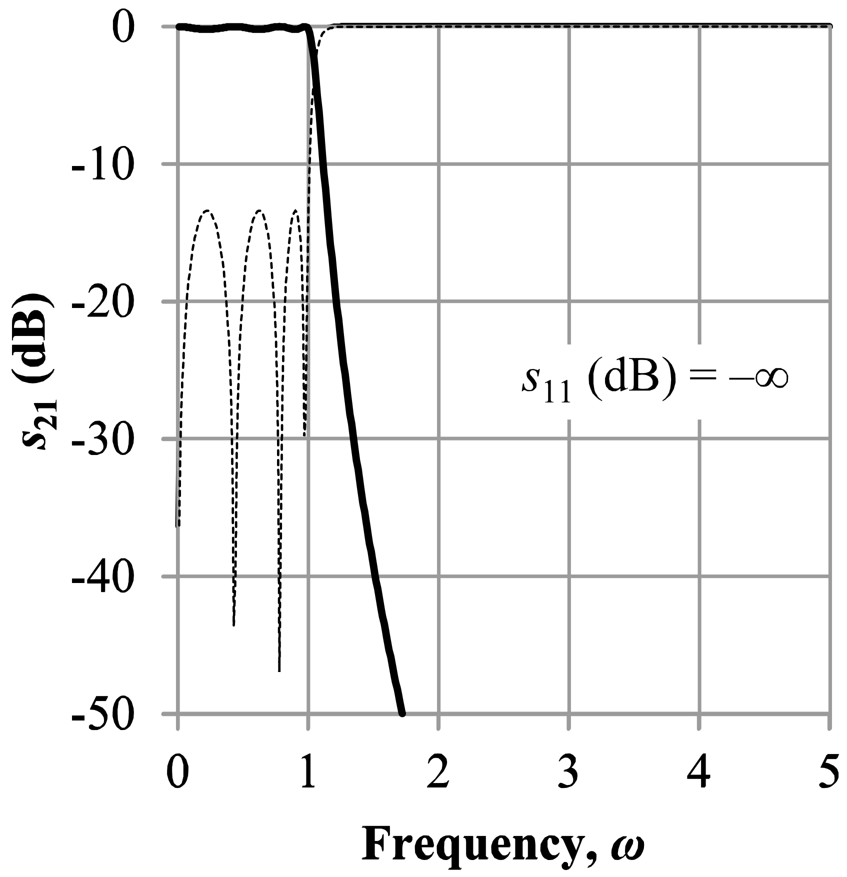
Type-II to Type-I Transformation

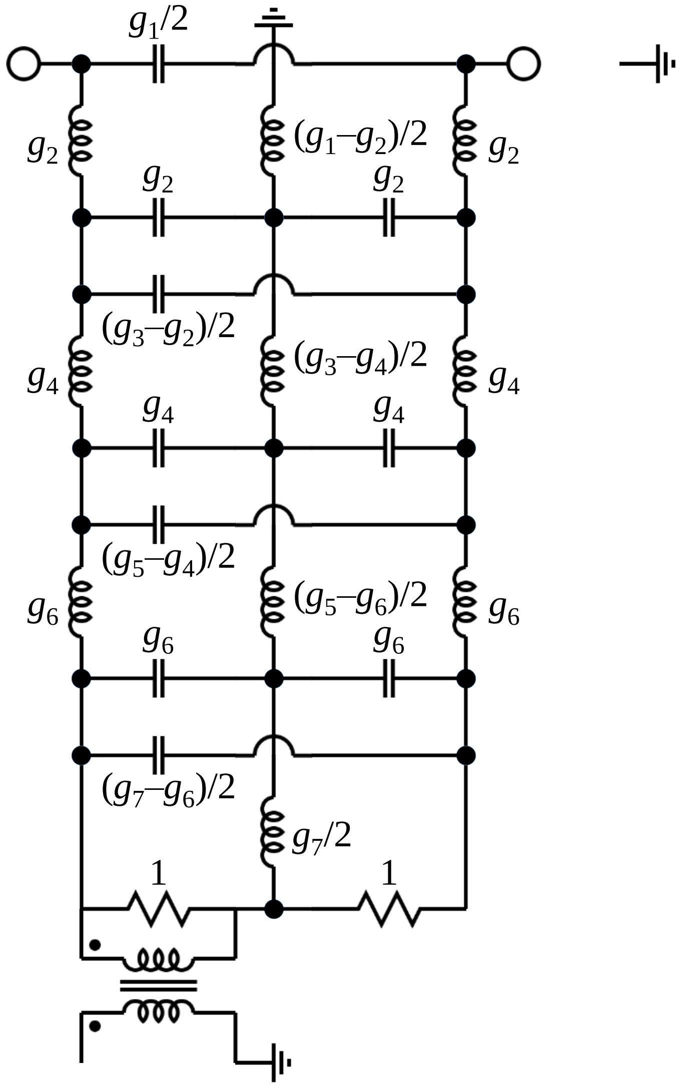


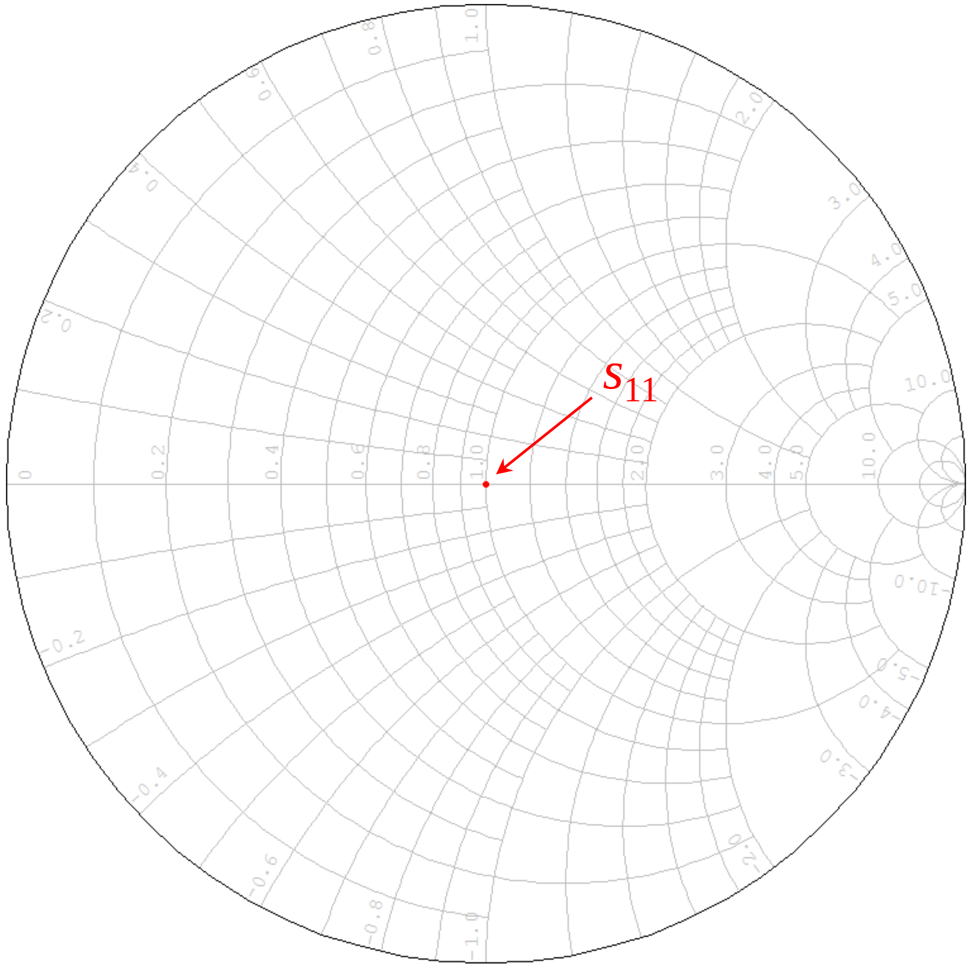




Type-II to Type-I Transformation

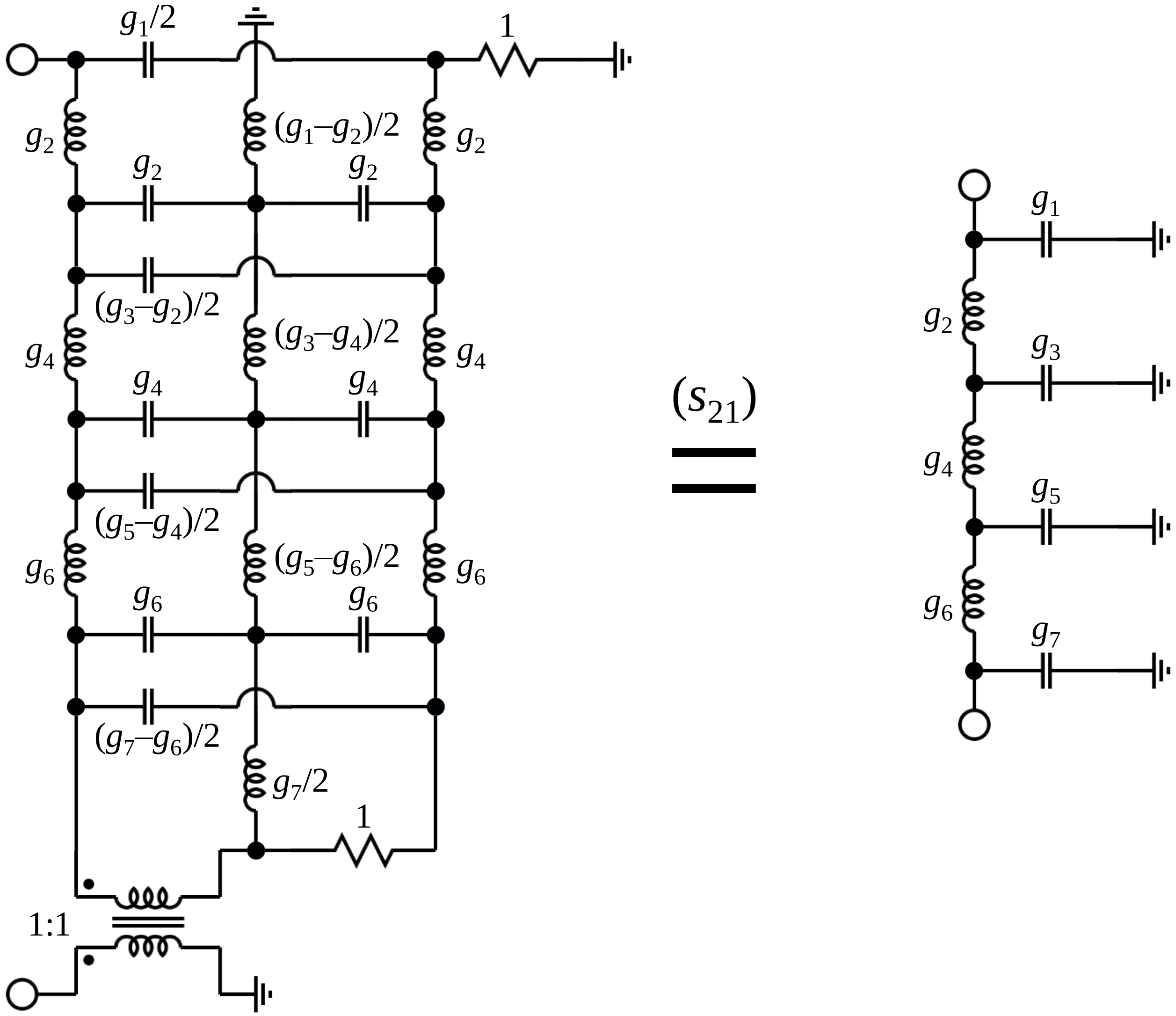






Type-I Transmission Equivalent to Standard Ladder

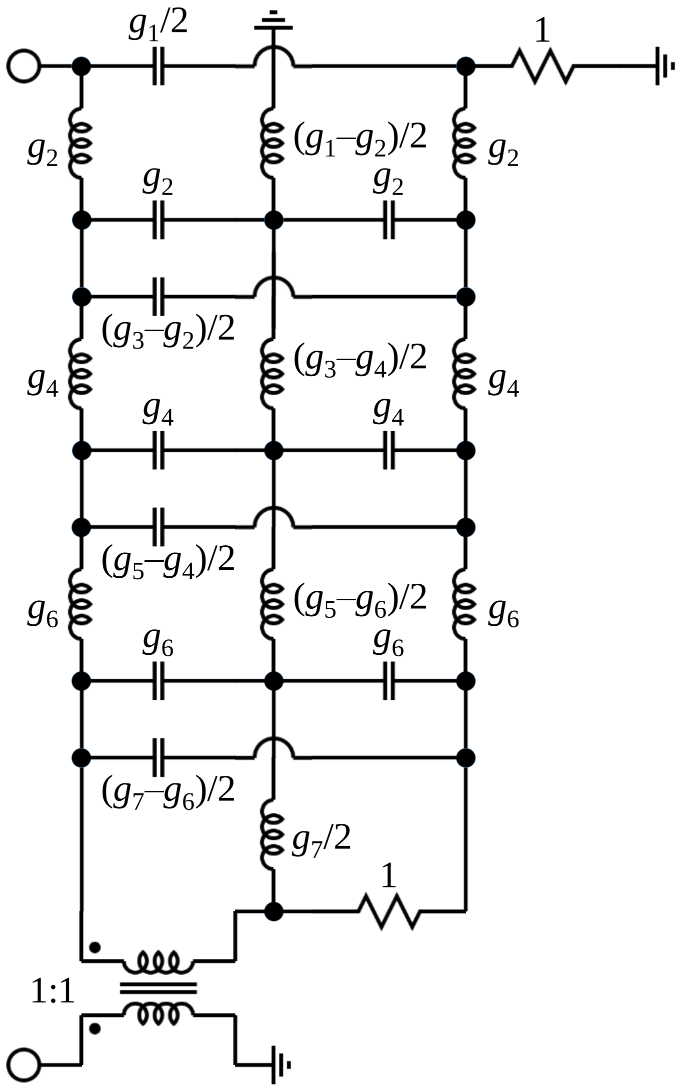
(but Reflectionless)



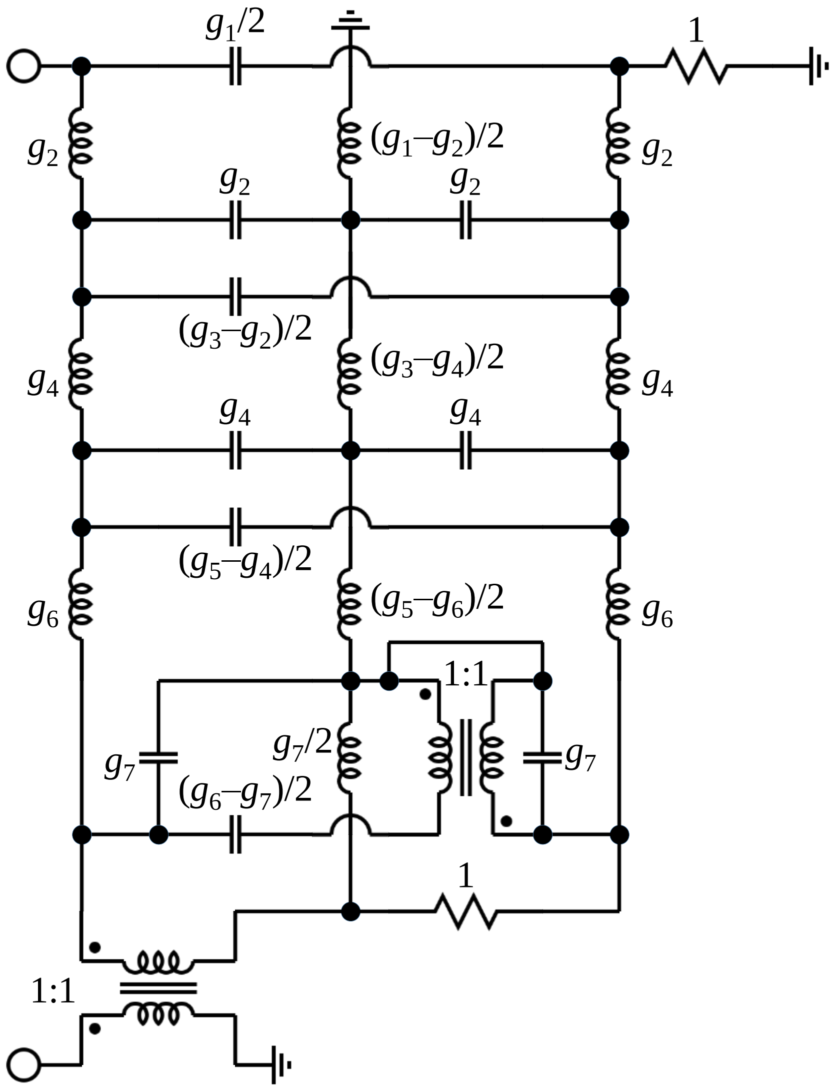
Designation	ε	g_1	g_2	g_3	g_4	g_5	g_6	g_7
Butterworth	0	0.445	1.247	1.802	2.000	1.802	1.247	0.445
Chebyshev	0.0100	0.535	1.179	1.464	1.500	1.464	1.179	0.535
	0.0189	0.621	1.274	1.569	1.569	1.569	1.274	0.621
	0.0500	0.807	1.396	1.757	1.634	1.757	1.396	0.807
	0.1000	1.008	1.437	1.940	1.622	1.940	1.437	1.008
	0.1500	1.173	1.424	2.089	1.576	2.089	1.424	1.173
	0.2000	1.323	1.392	2.229	1.520	2.229	1.392	1.323
	0.2187	1.377	1.377	2.280	1.498	2.280	1.377	1.377
	0.2500	1.465	1.350	2.366	1.461	2.366	1.350	1.465
Legendre	~	1.864	1.590	2.151	1.727	1.939	1.477	0.839

Negative Elements Removed As-Needed

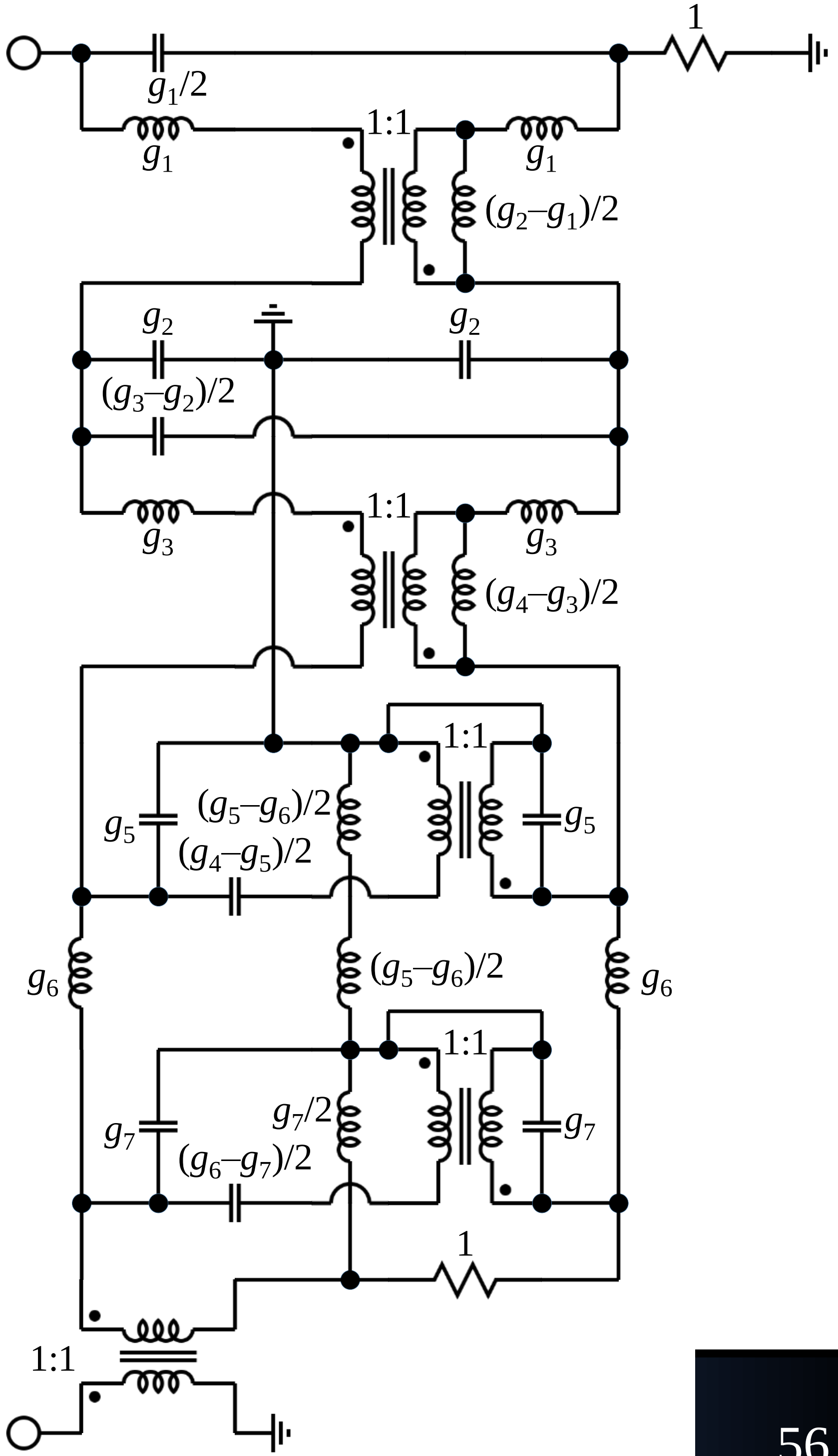
Chebyshev-I ($\varepsilon \geq 0.2187$)



Legendre

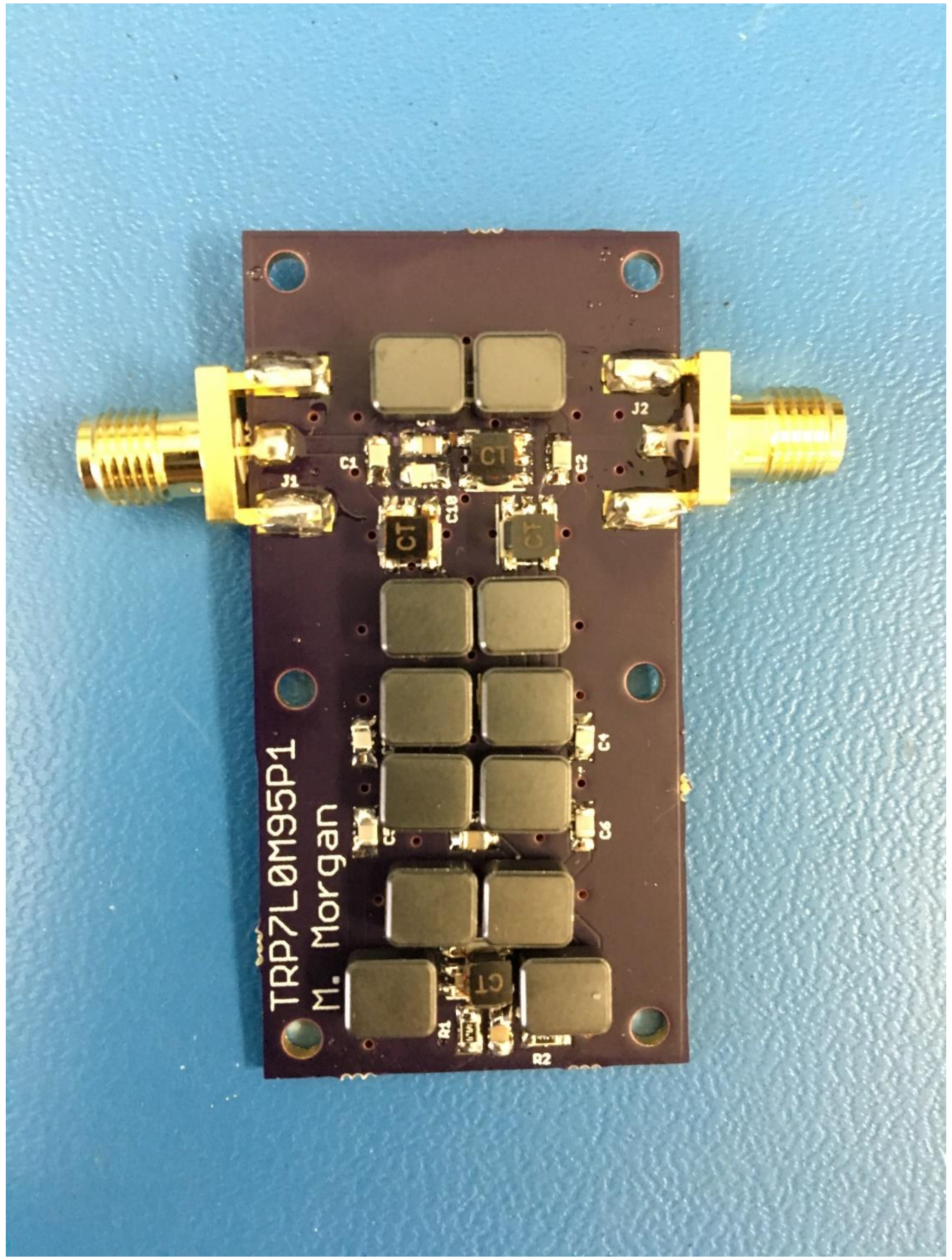


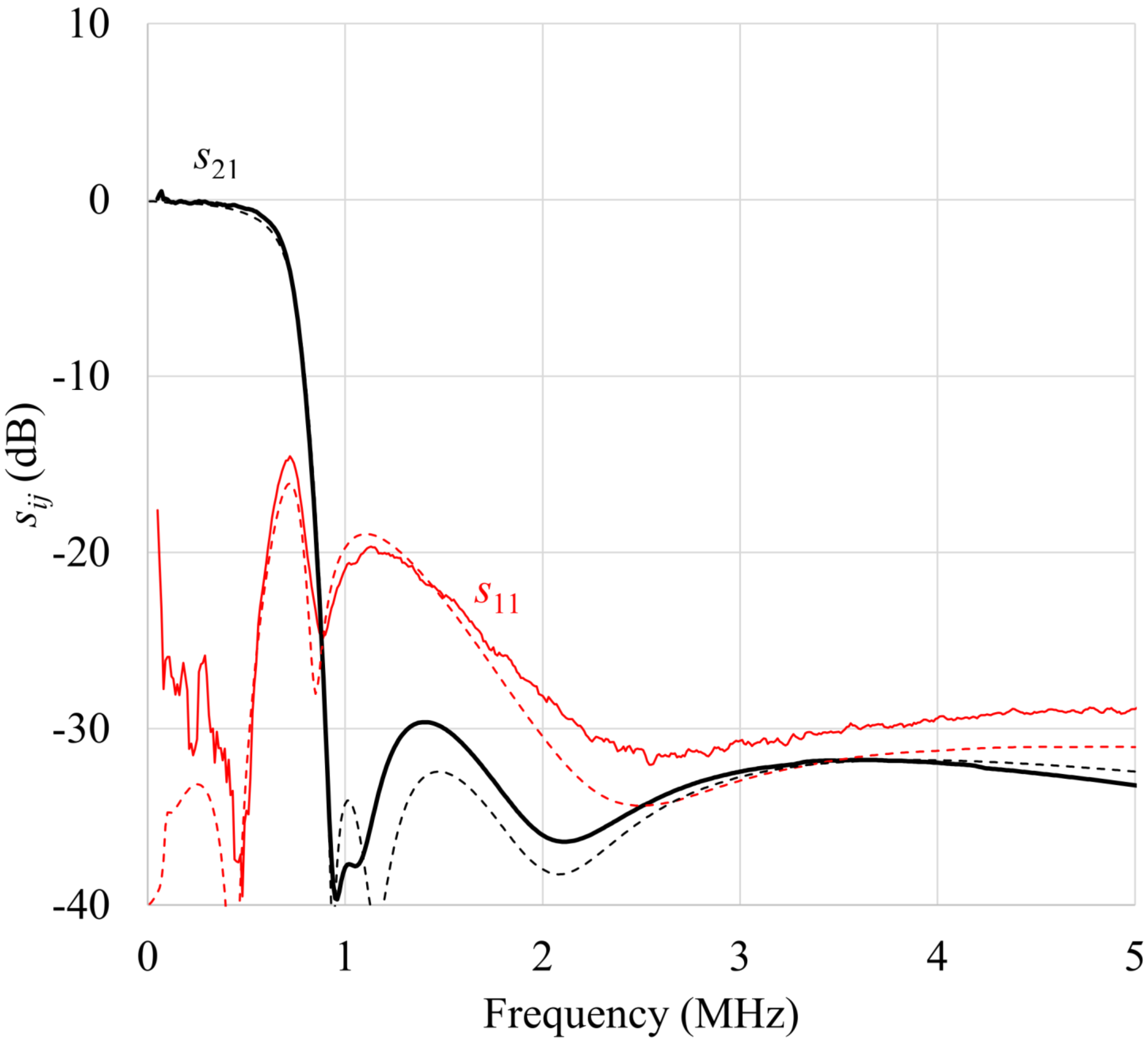
Butterworth



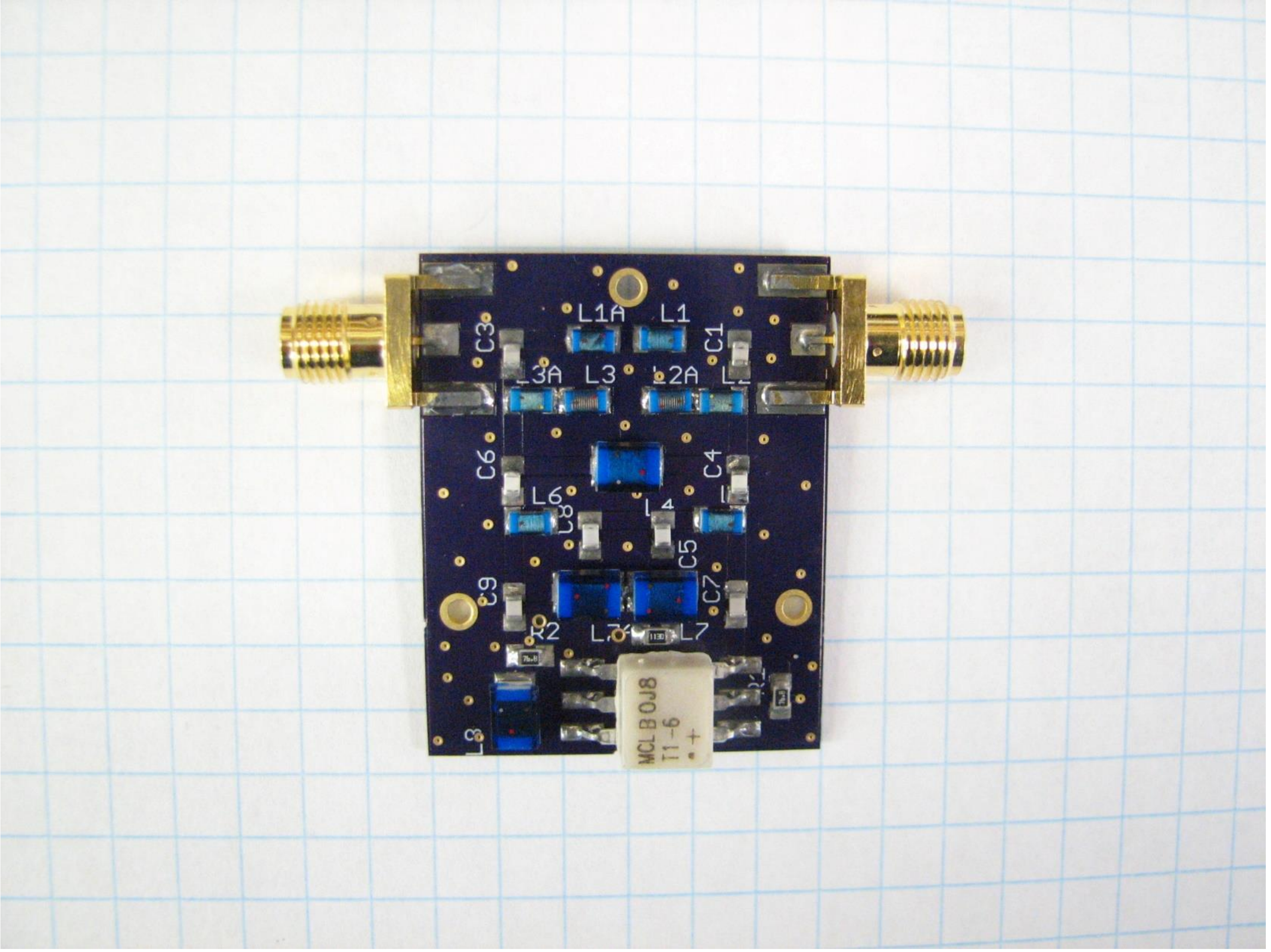
Implementation: Discrete SMT

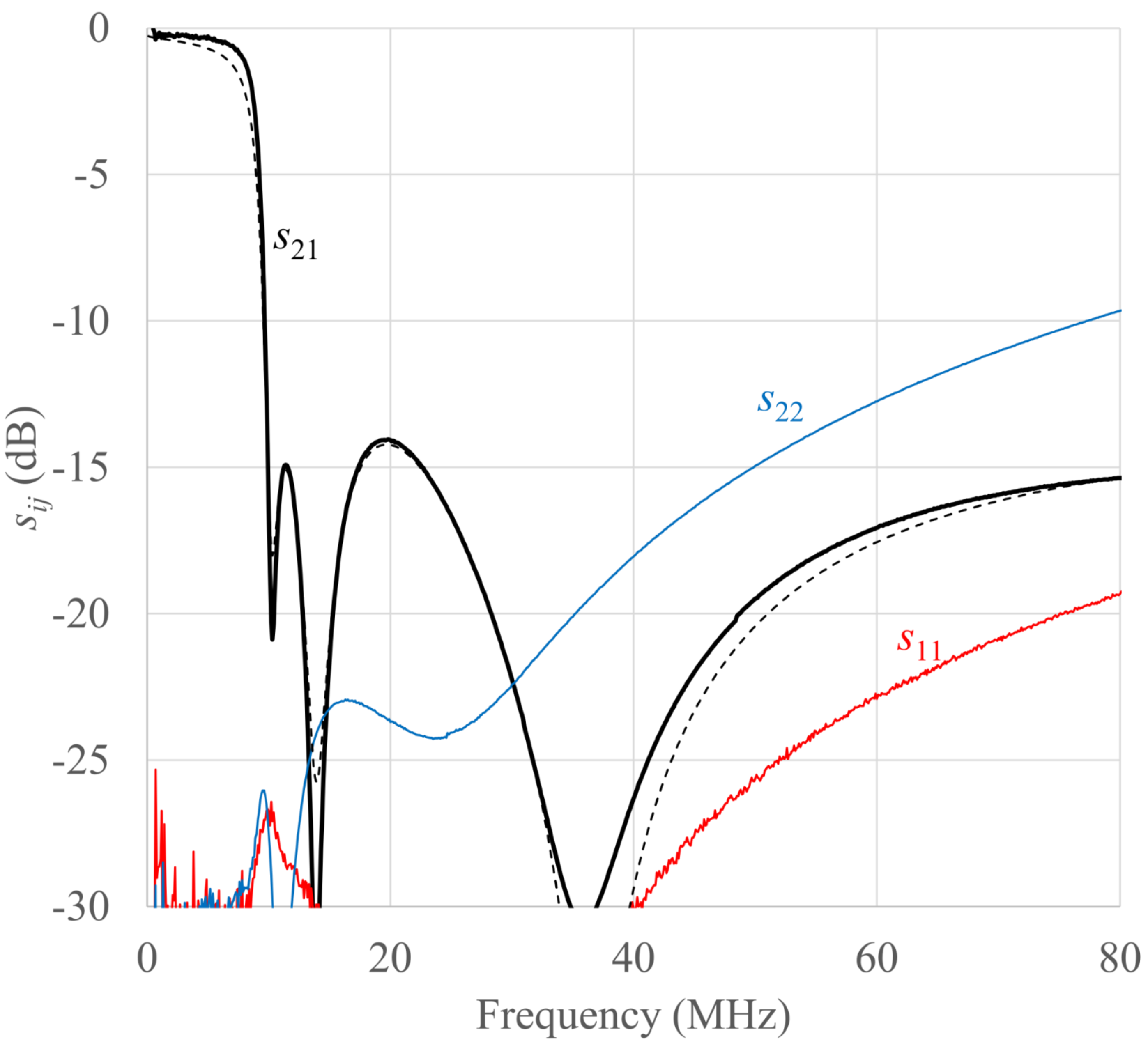
Deep Rejection Chebyshev Type-II Filter (*N*=7)



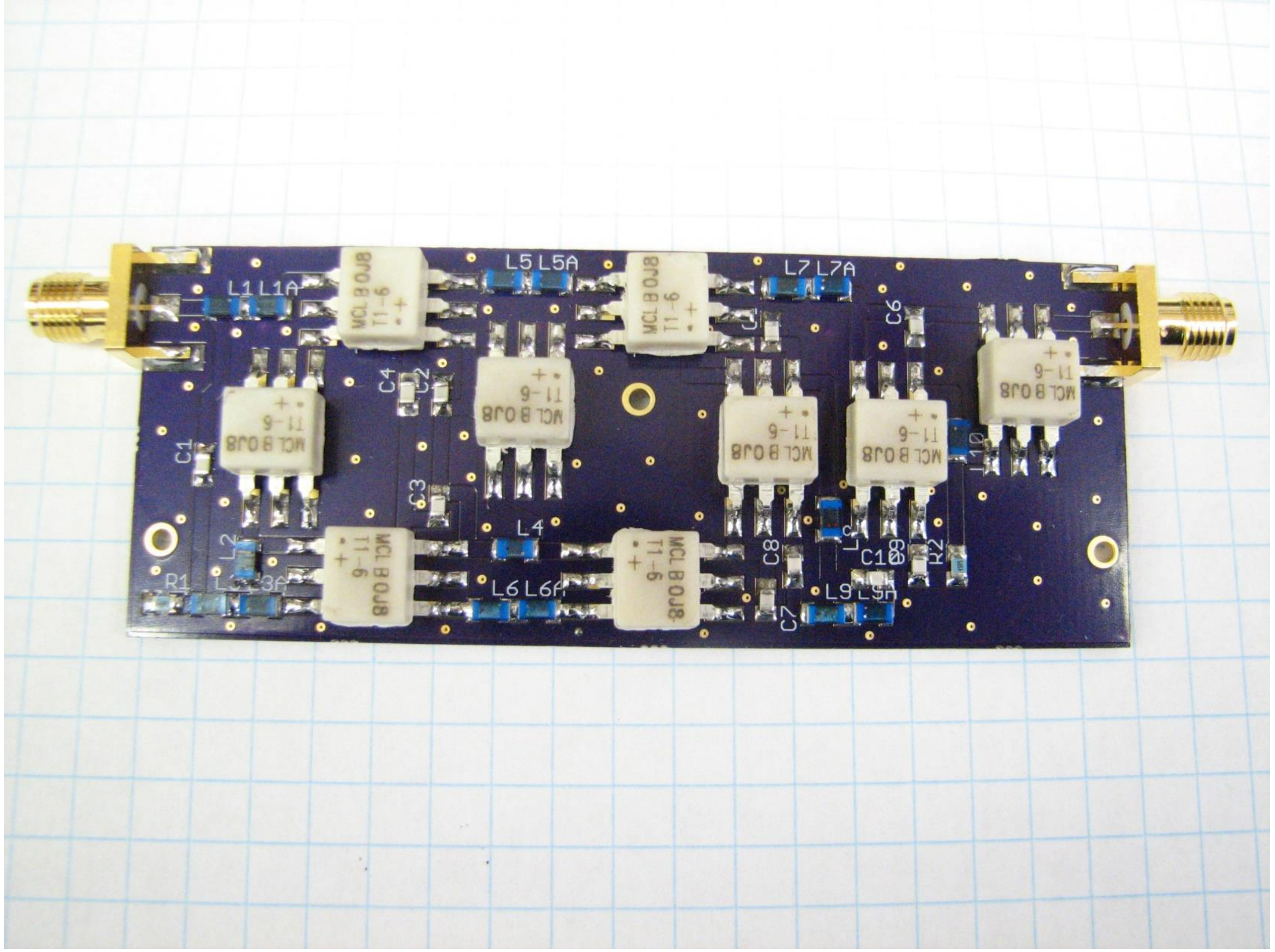


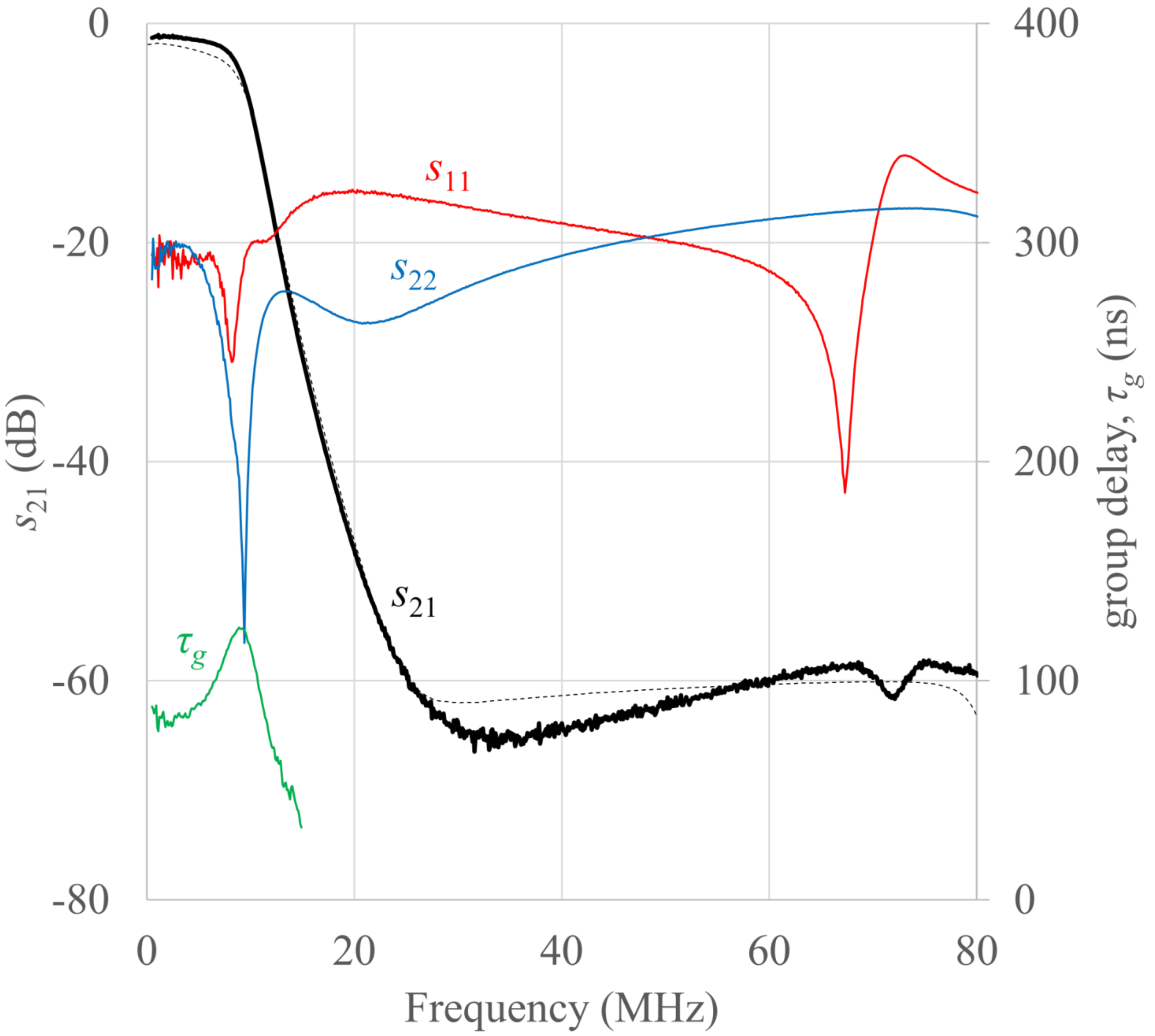
Chebyshev Type-II Even-Order Filter (N=6)



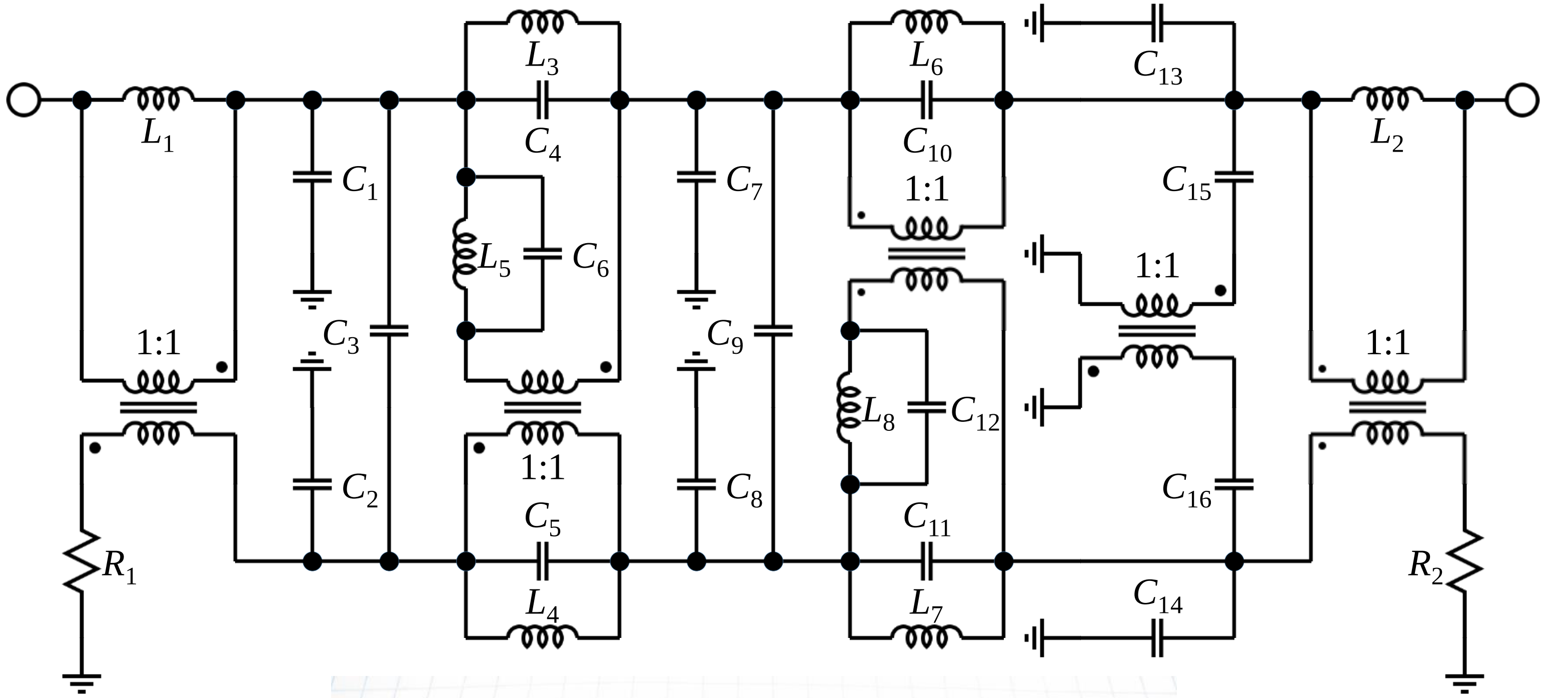


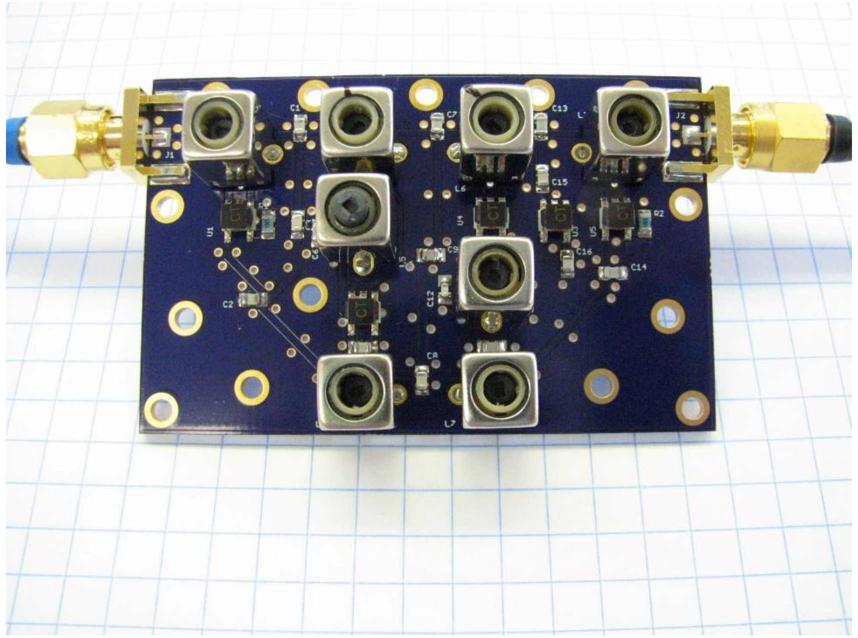
Butterworth Filter (N=7)

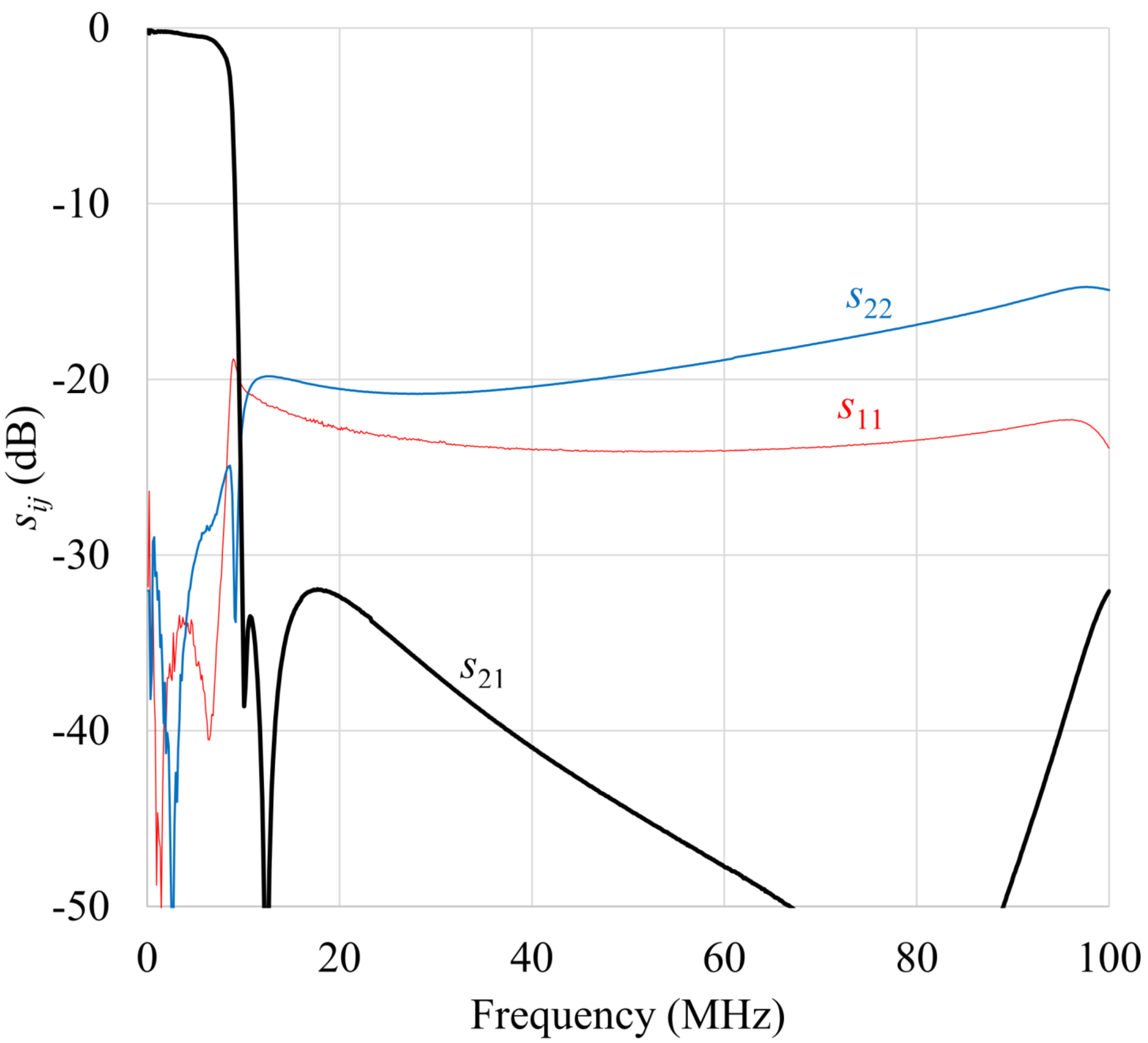




Elliptic Filter



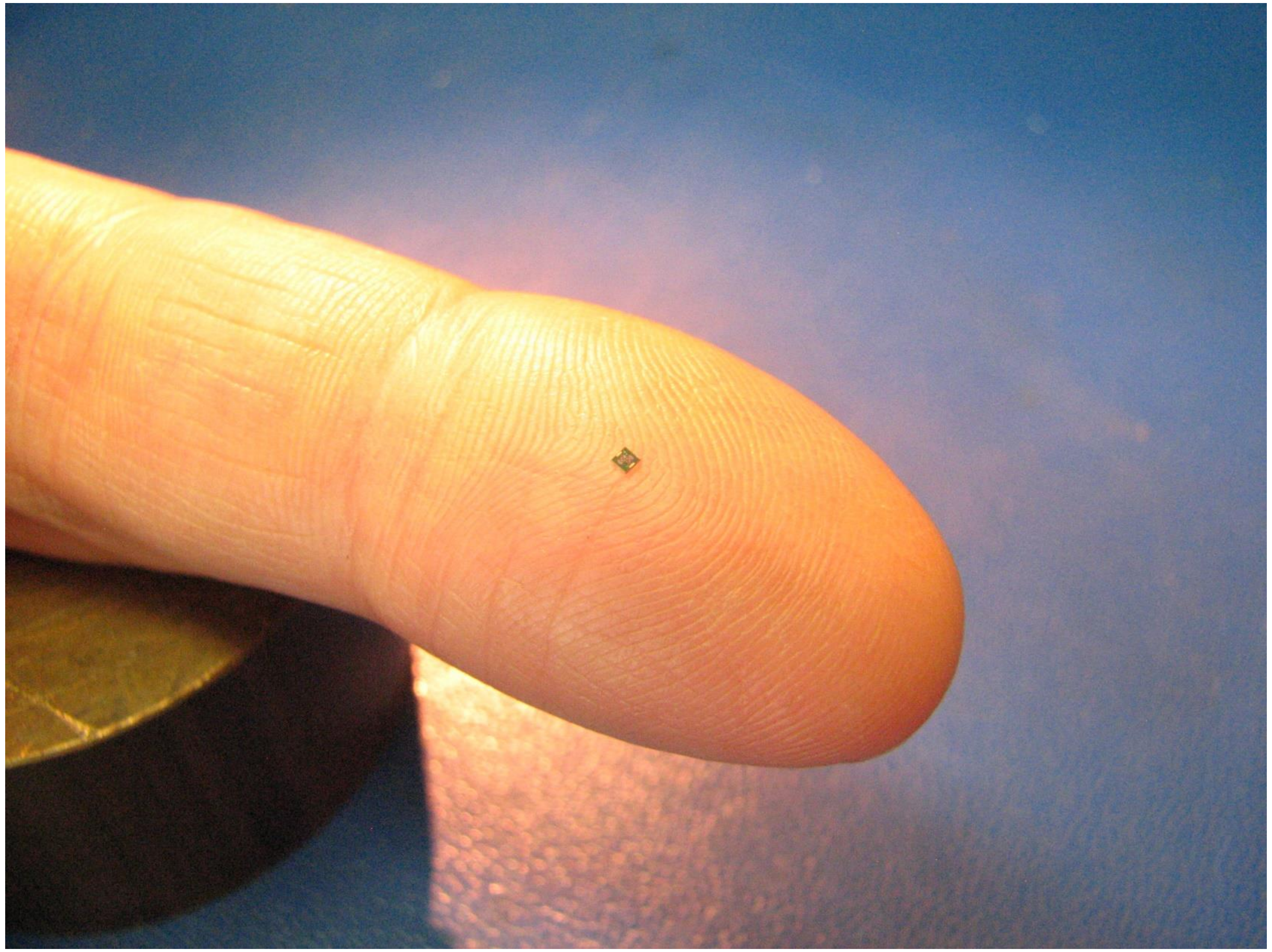


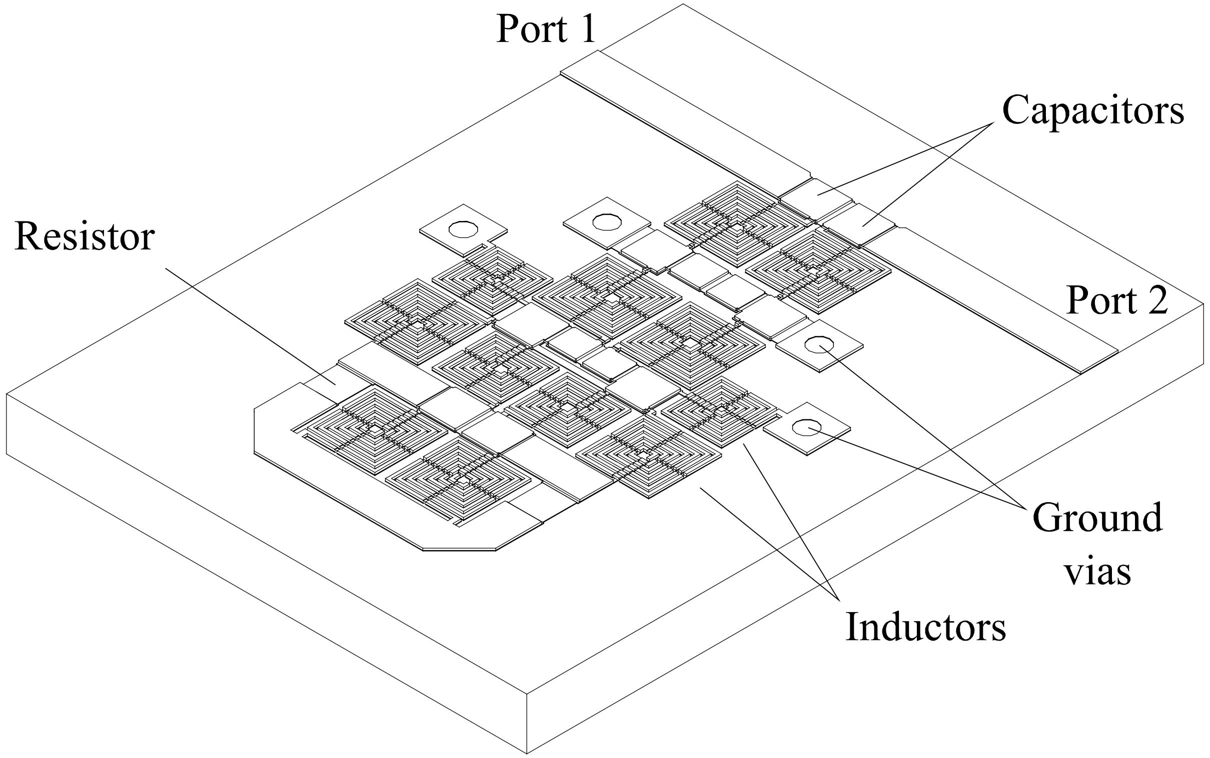


A. Guilabert, M. Morgan, and T. Boyd, "Reflectionless filters for generalized elliptic transmission functions," *IEEE Transactions on Circuits and Systems I*, vol. 66, no. 12, pp. 4606-4618, December 2019.

Implementation: Monolithic Die

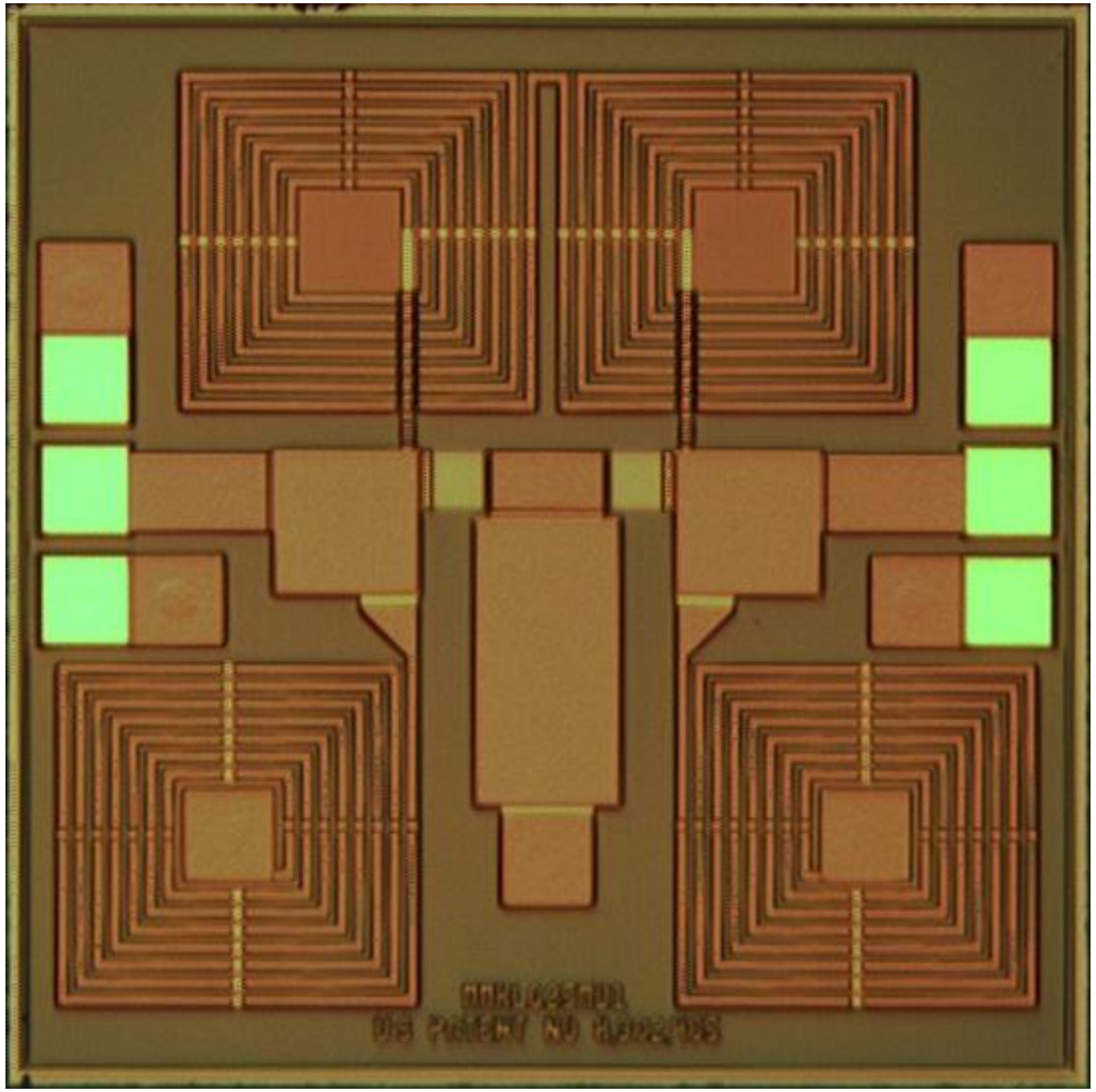
Most Compact Implementation



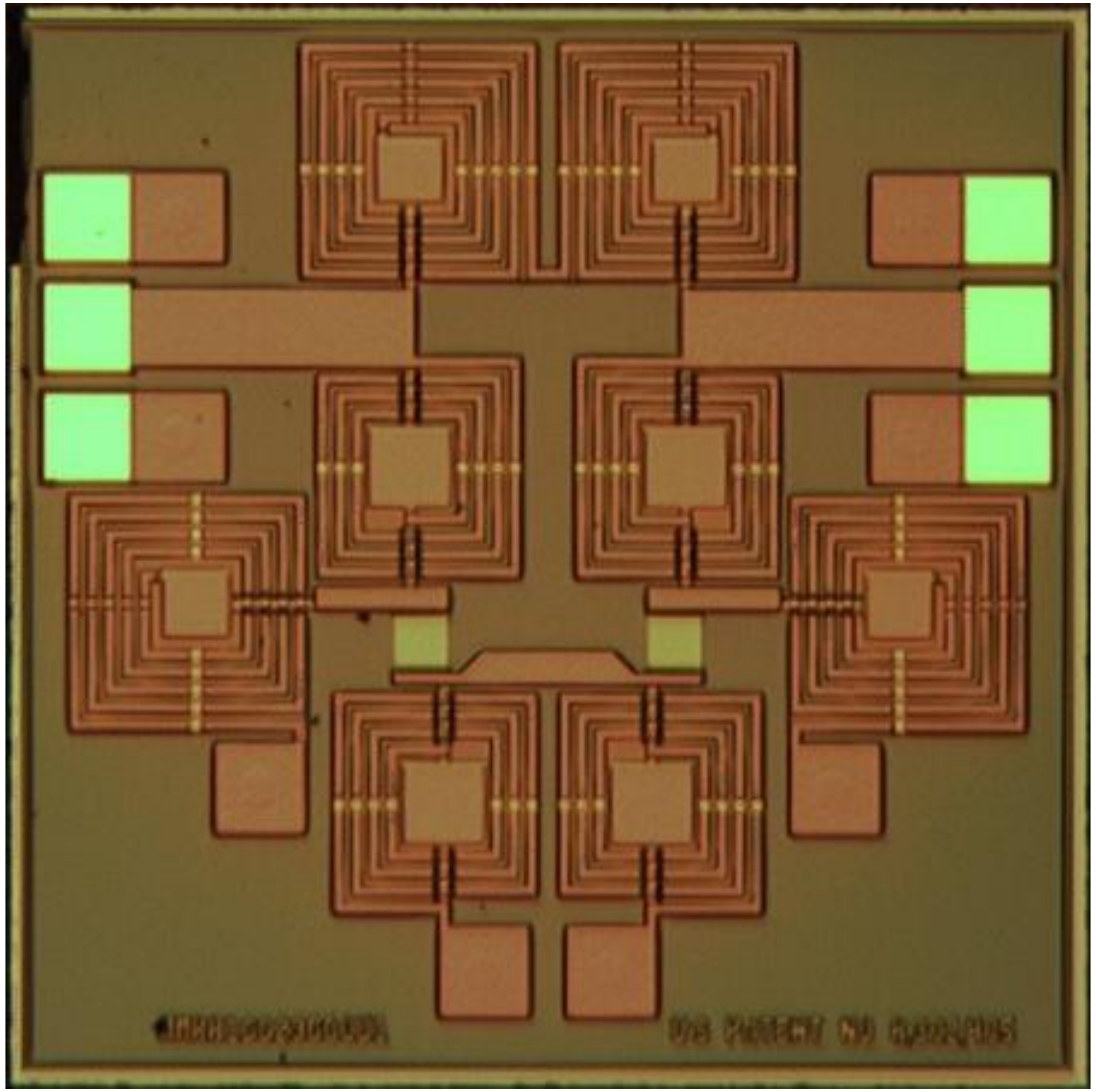


GaAs Integrated Passive Device (IPD) Fabrication

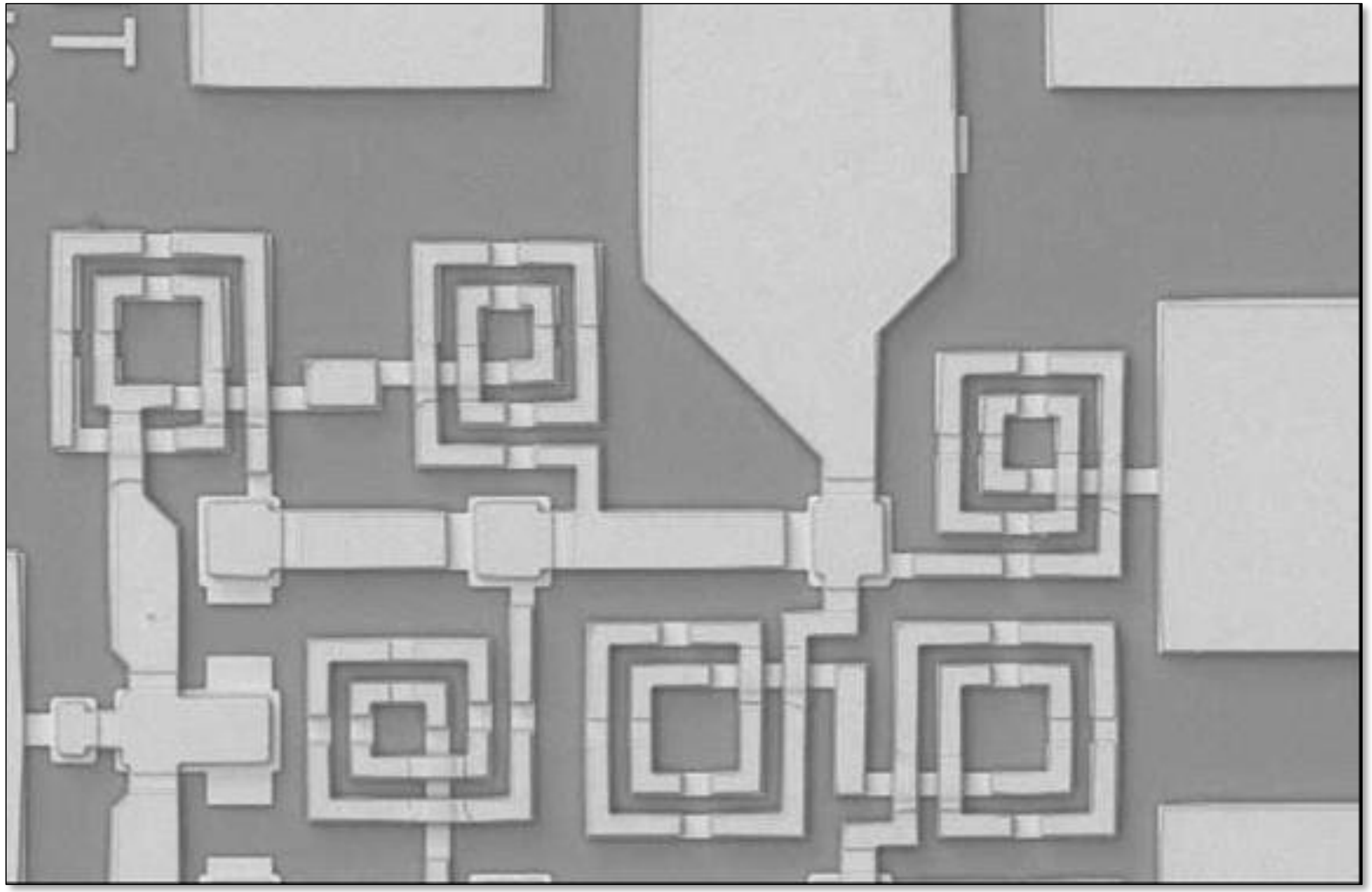
625 MHz Low-Pass Filter

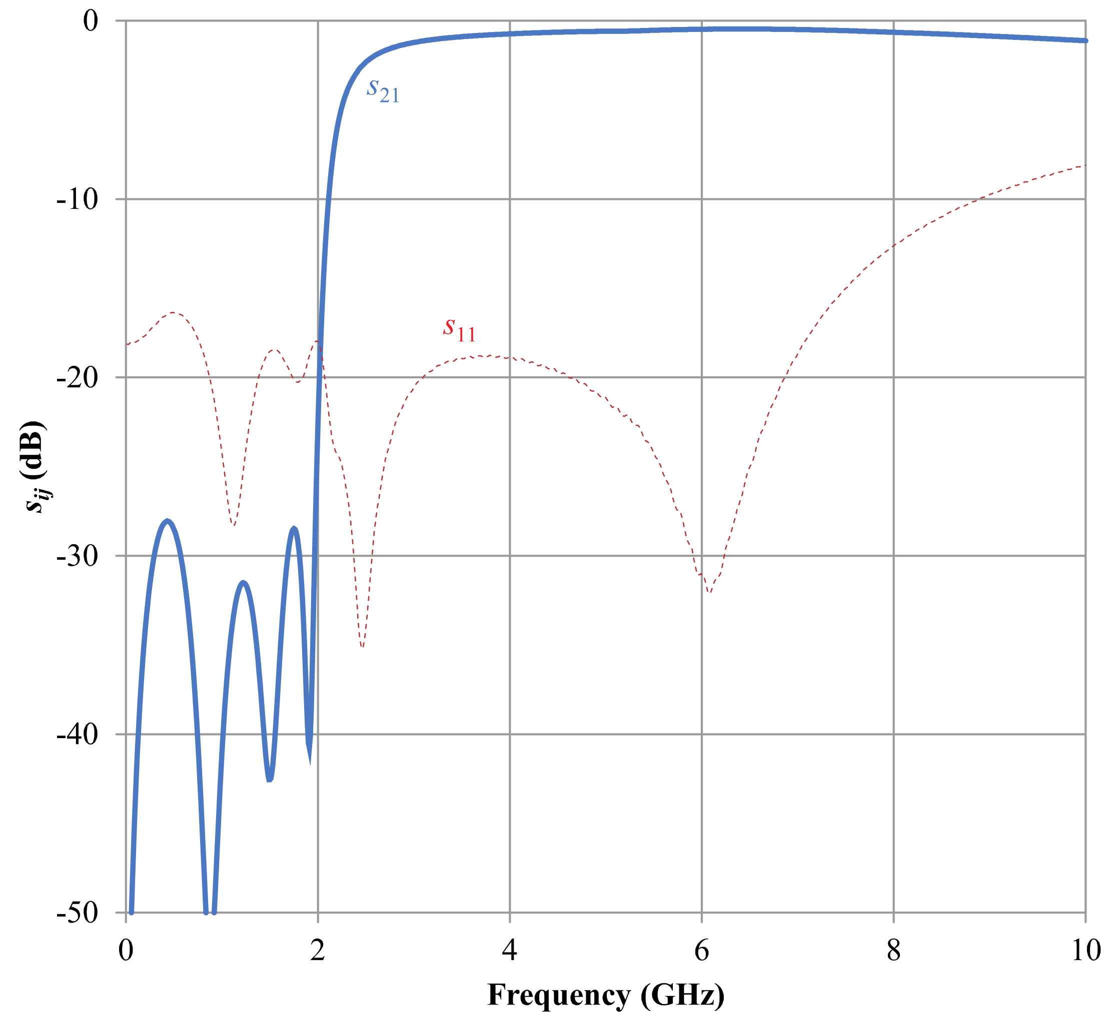


1.6–3.6 GHz Band-Pass Filter

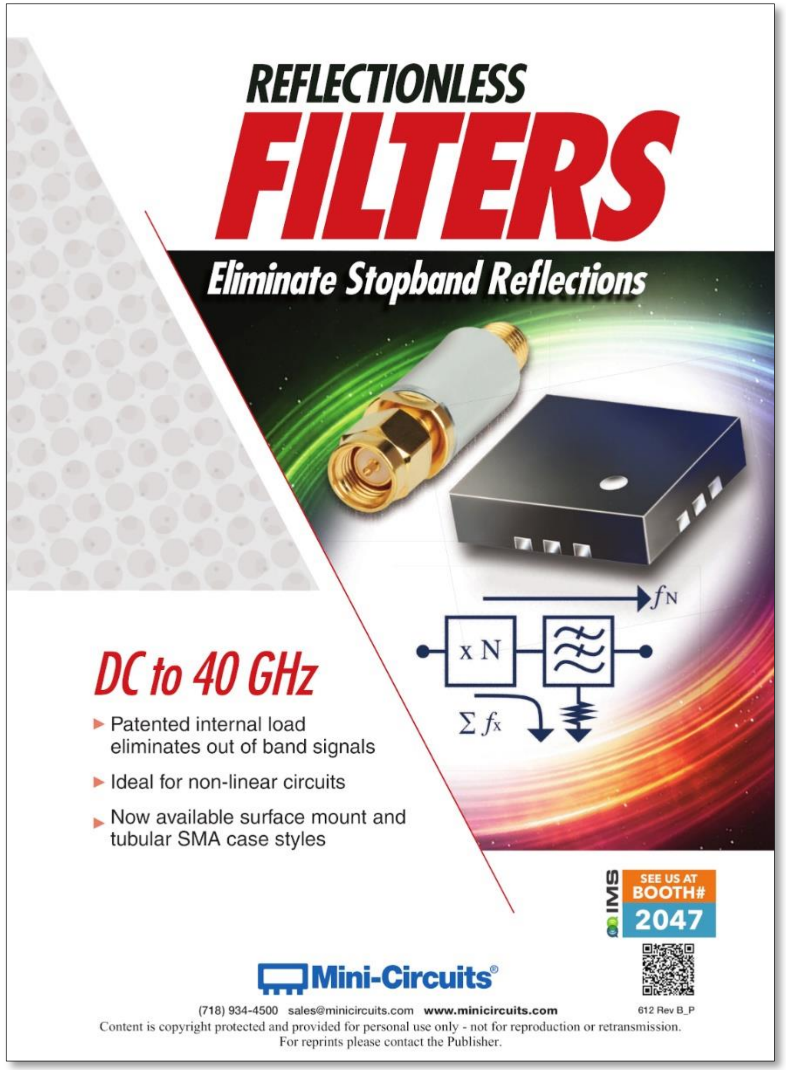


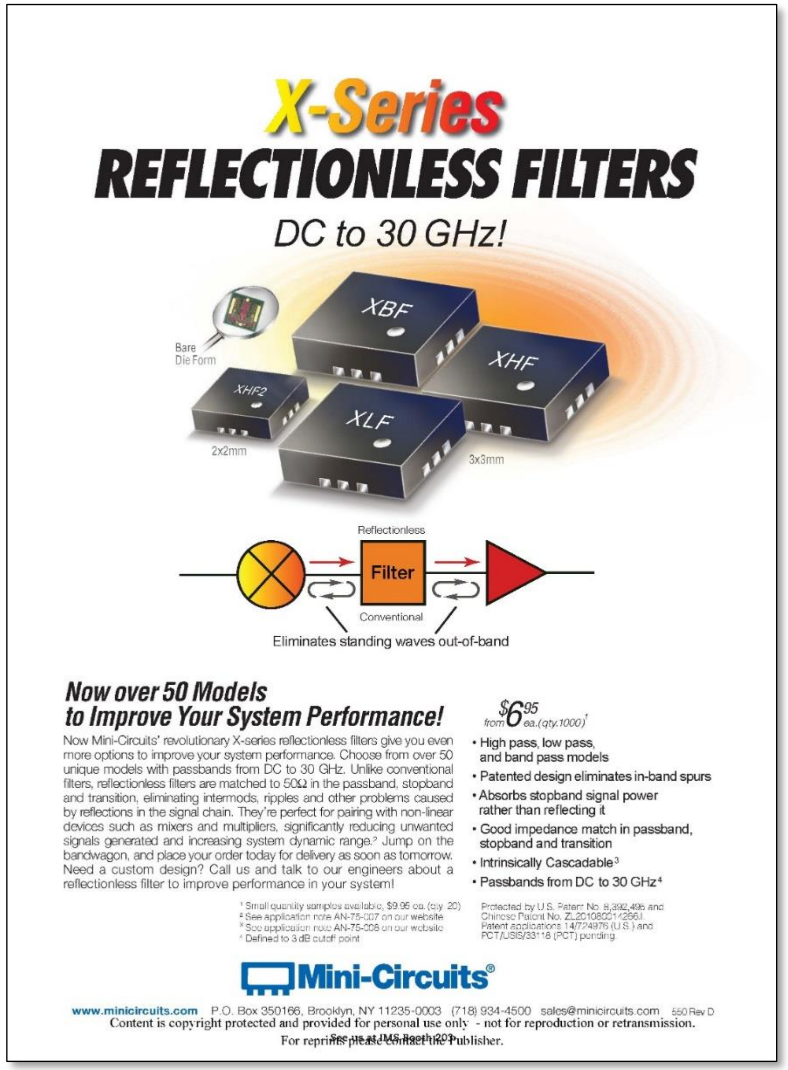
GaAs IPD High-Pass Filter (N=7)





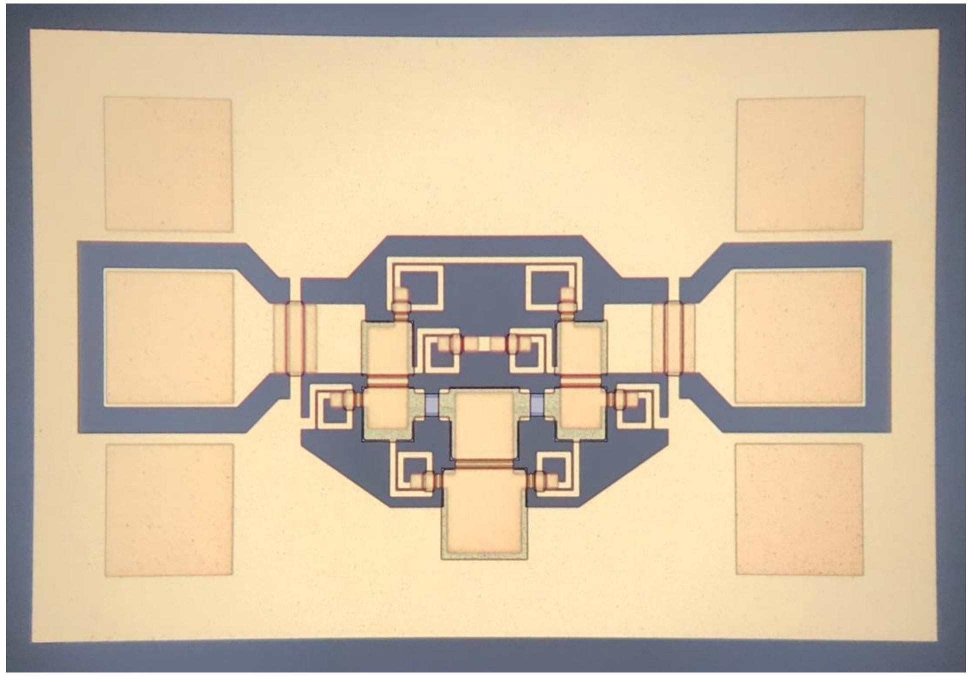
Now Marketed by Mini-Circuits



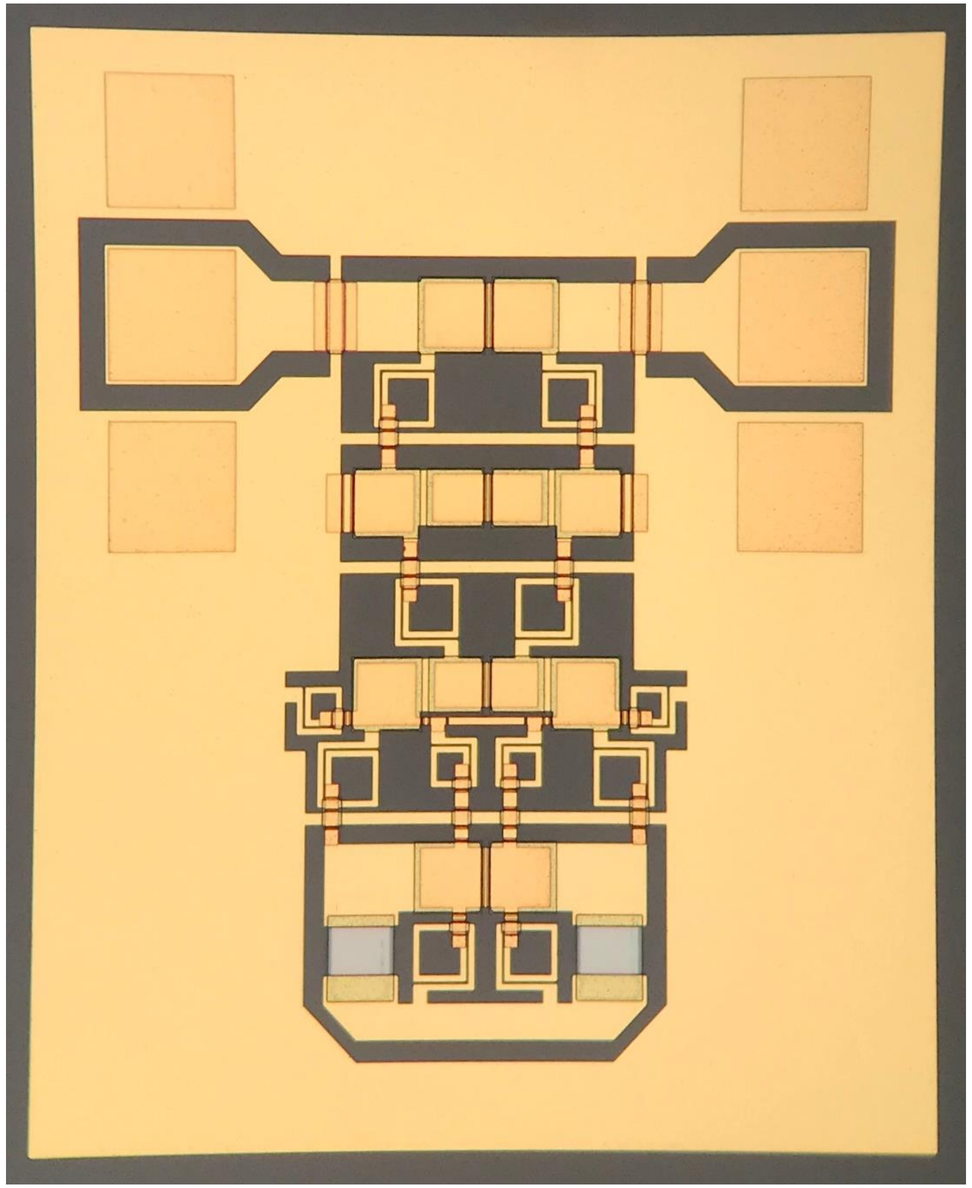


Thin-Film on Quartz

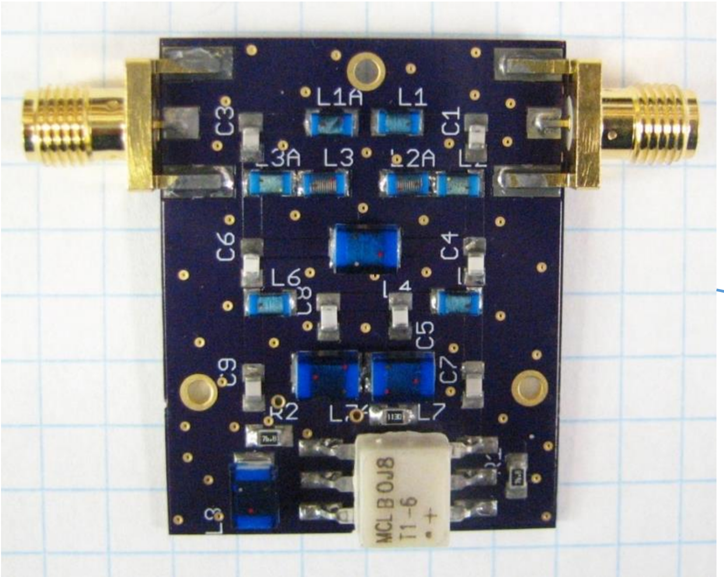
60 GHz Low-Pass Filter (N=5)

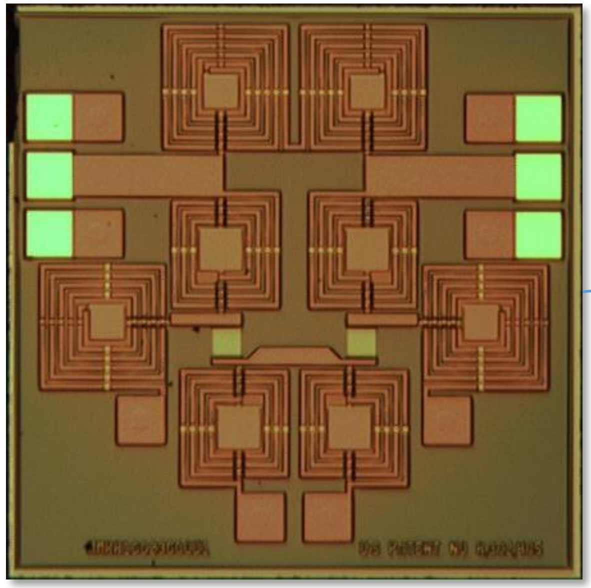


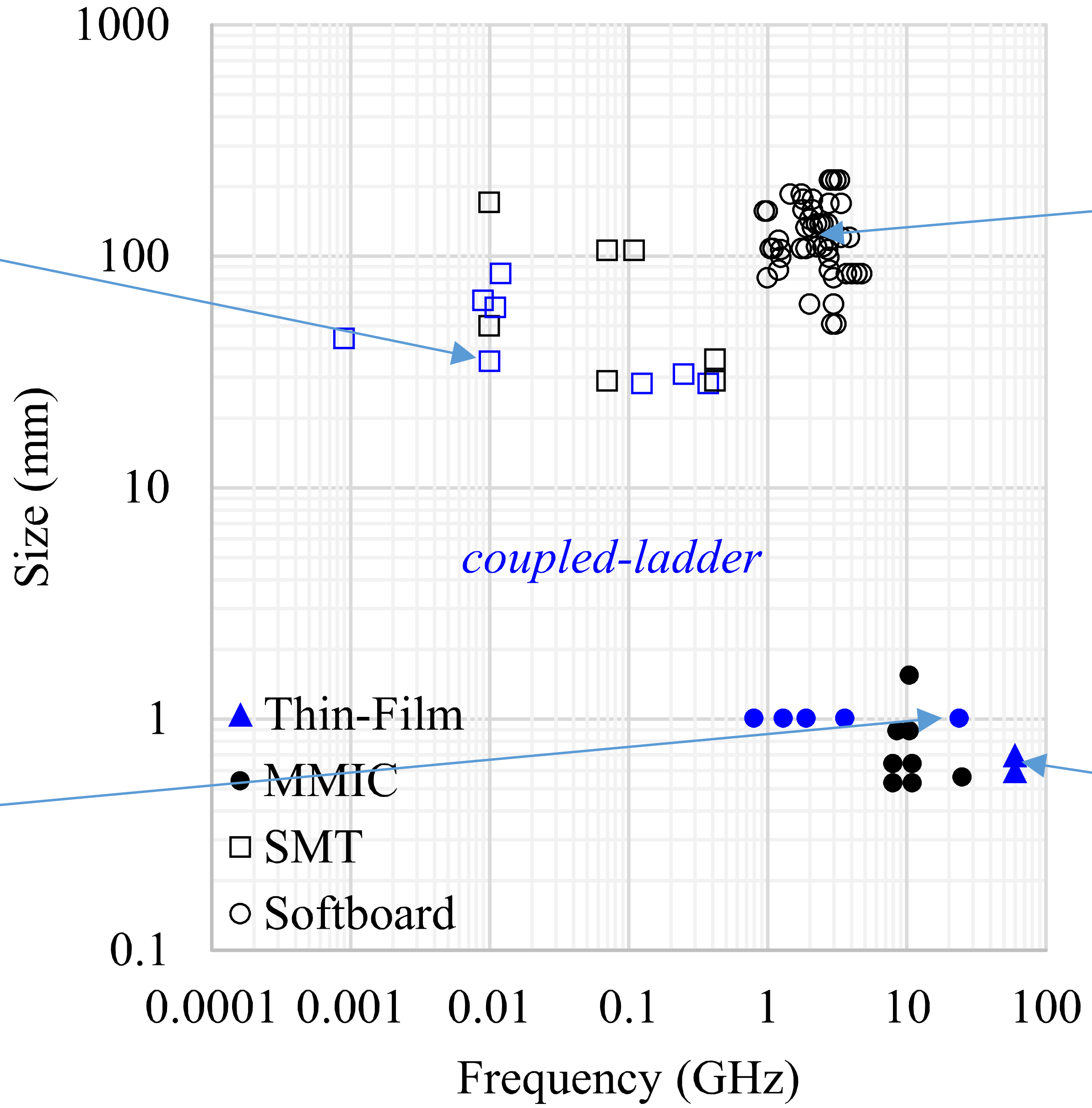
60 GHz High-Pass Filter (N=7)

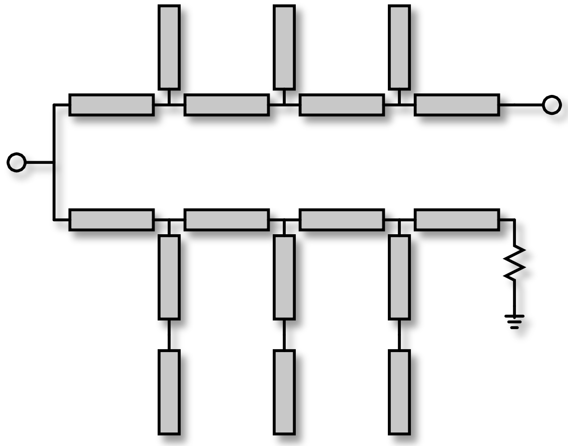


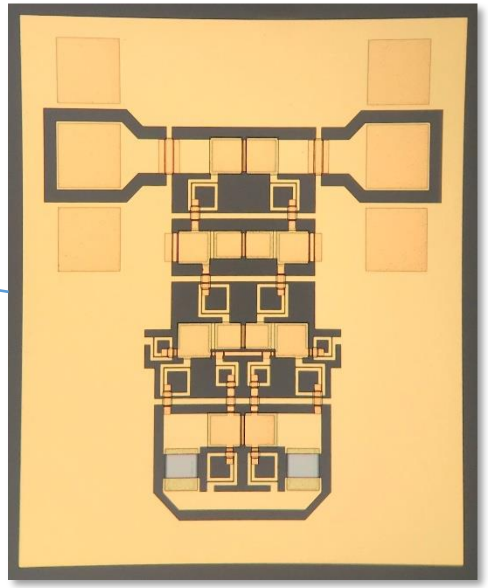
Size and Frequency of Various Implementations











References

Books and Book Chapters

- M. Morgan, *Reflectionless Filters*, Norwood, MA: Artech House, January 2017.
- M. Morgan, “Reflectionless filter topologies,” *Wiley Encyclopedia of Electrical and Electronics Engineering*, pp. 1-13, August 2019.
- M. Morgan, “Planar reflectionless filters.” In J. Hong, ed., *Advances in Planar Filters Design*, London: SciTech Publishing, 2019.

Academic Papers and Magazine Articles

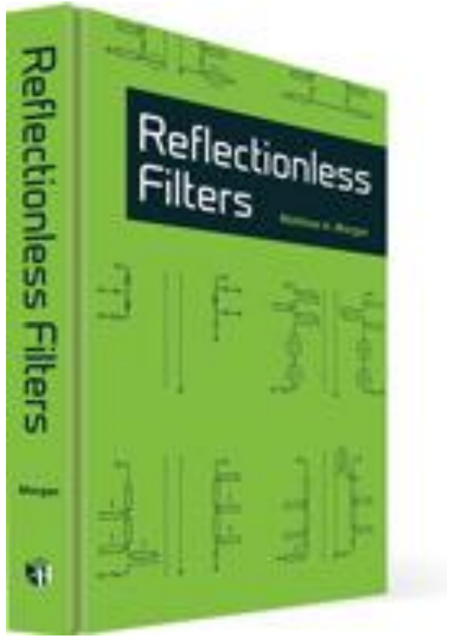
- A. Guilabert, M. Morgan, and T. Boyd, “Reflectionless filters for generalized elliptic transmission functions,” *IEEE Transactions on Circuits and Systems I*, vol. 66, no. 12, pp. 4606-4618, December 2019.
- R. Shrotriya and M. Morgan, “Filtering without reflections: flattening multiplier chain conversion efficiency & more,” *Microwave Journal White Paper*, September 2019.
- M. Morgan, W. Groves, and T. Boyd, “Reflectionless filter topologies supporting arbitrary low-pass ladder prototypes,” *IEEE Transactions on Circuits and Systems I*, vol. 66, no. 2, pp. 594-604, February 2019.
- M. Morgan, “Think outside the band: design and miniaturization of absorptive filters,” *IEEE Microwave Magazine*, vol. 19, no. 7, pp. 54-62, November 2018.
- M. Morgan, “A better way to filter -- part I,” *NRAO Blog*, March 29, 2018.
- R. Setty, B. Kaplan, M. Morgan, and T. Boyd, “Combining MMIC reflectionless filters to create UWB bandpass filters,” *Microwave Journal*, vol. 61, no. 3, pp. 60-72, March 2018.
- M. Morgan and T. Boyd, “Reflectionless filter structures,” *IEEE Trans. Microw. Theory Techn.*, vol. 63, no. 4, pp. 1263-1271, April 2015.
- M. Morgan and T. Boyd, “Theoretical and experimental study of a new class of reflectionless filter,” *IEEE Trans. Microwave Theory Tech.*, vol. 59, no. 5, pp. 1214-1221, May 2011.

Application Notes

- “Advantages of cascading reflectionless filters,” Mini-Circuits Application Note, AN75-008.
- “Pairing mixers with reflectionless filters to improve system performance,” Mini-Circuits Application Note, AN75-007.

Patents

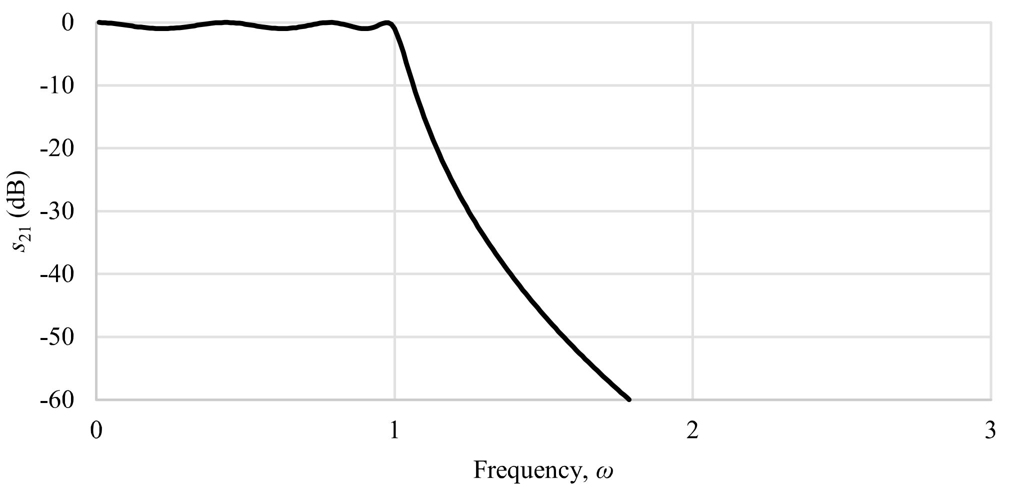
- M. Morgan, “Deep rejection reflectionless filters,” *U.S. Patent No. 10,530,321*, January 7, 2020, *Taiwan Patent No. I699970B*, July 21, 2021.
- M. Morgan, “Sub-network enhanced reflectionless filter topology,” *U.S. Patent No. 9,705,467*, July 11, 2017, *No. 10,230,348*, March 12, 2019, *Taiwan Patent No. I581494B*, May 1, 2017, *People’s Republic of China Patent No. 107078708B*, February 9, 2021.
- M. Morgan, “Optimal response reflectionless filter topologies,” *U.S. Patent No. 10,516,378*, December 24, 2019.
- M. Morgan, “Optimal response reflectionless filters,” *U.S. Patent No. 10,374,577*, August 6, 2019, *No. 10,263,592*, April 16, 2019. *Taiwan Patent No. I653826B*, March 11, 2019.
- M. Morgan, “Transmission line reflectionless filters,” *U.S. Patent No. 9,923,540*, March 20, 2018, *No. 10,277,189*, April 30, 2019, *Japan Patent No. 6652970B2*, January 28, 2020, *6964152B2*, October 20, 2021.
- M. Morgan, “Reflectionless filters,” *U.S. Patent No. 8,392,495*, March 5, 2013. *People's Republic of China Patent No.102365784B*, July 30, 2014.



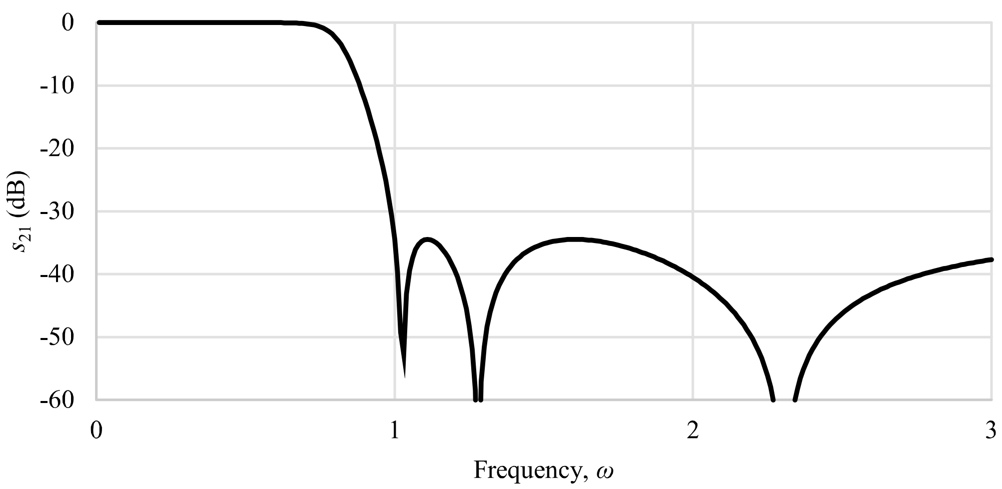
Backup Slides

Canonical Filter Responses

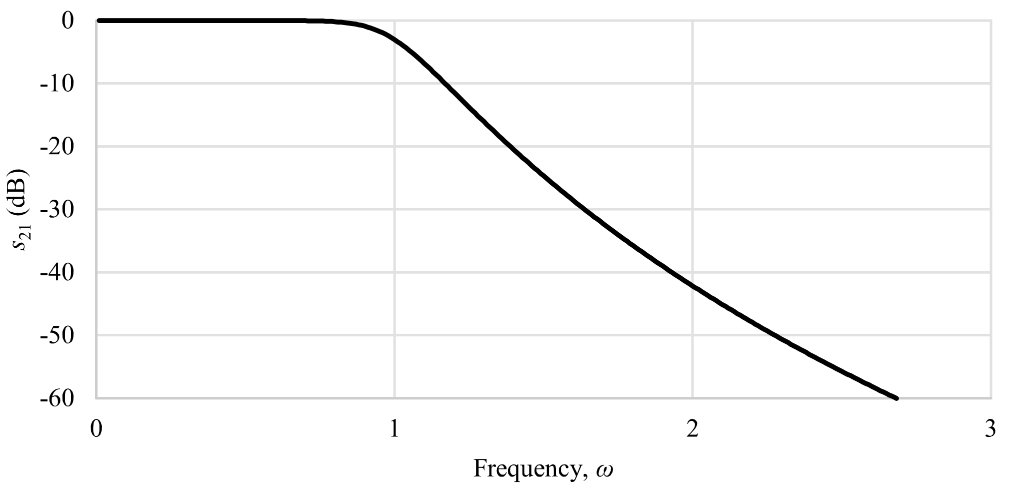
Chebyshev Type I



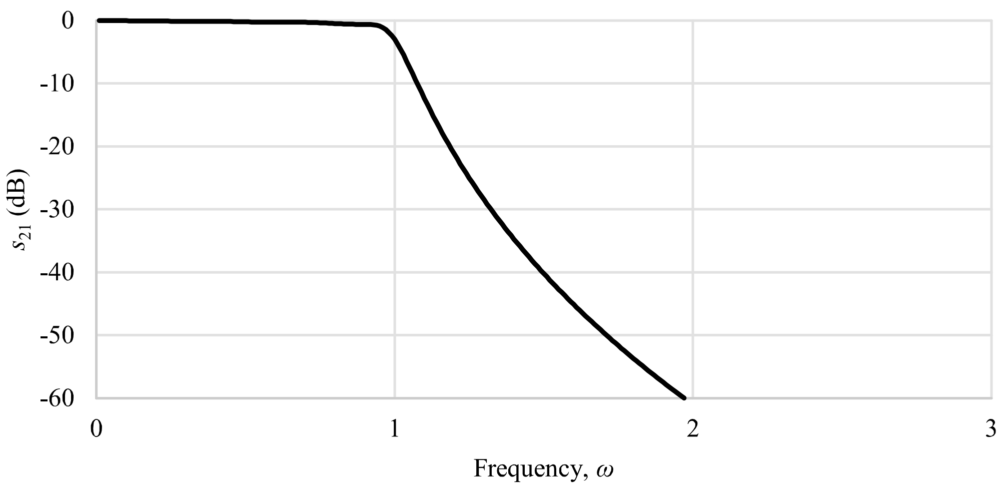
Chebyshev Type II



Butterworth

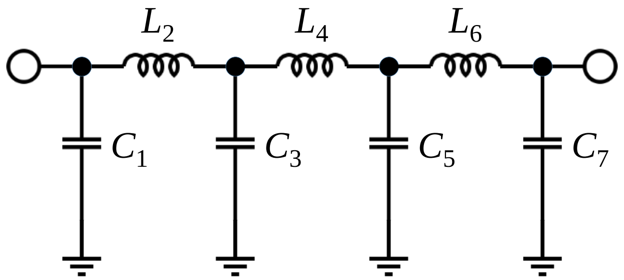


Legendre

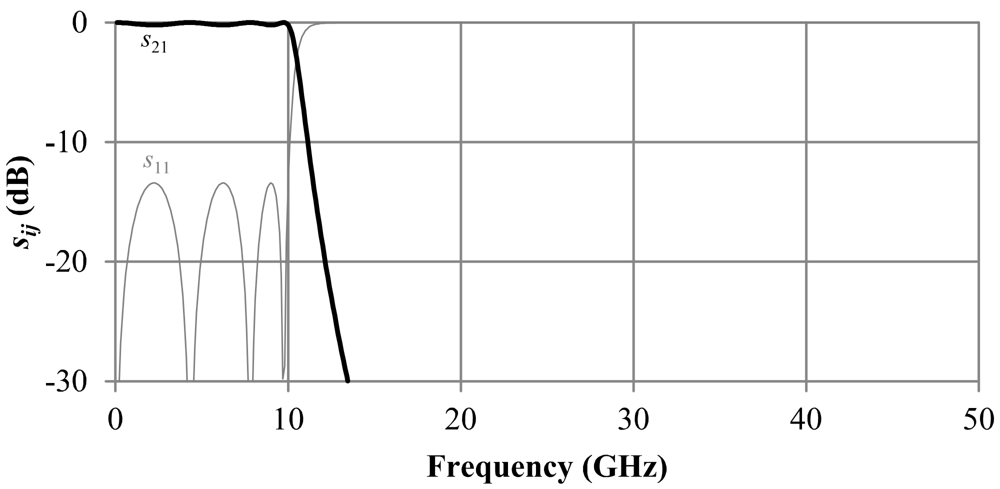


Conventional Topologies

Lumped-Element Ladder

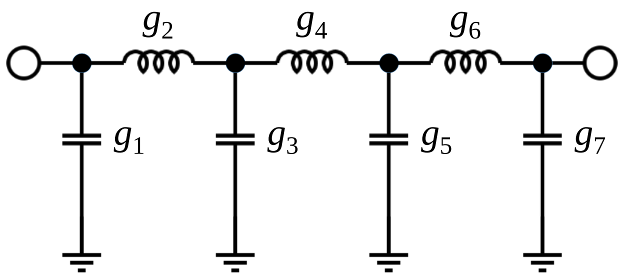


Real Frequency Response

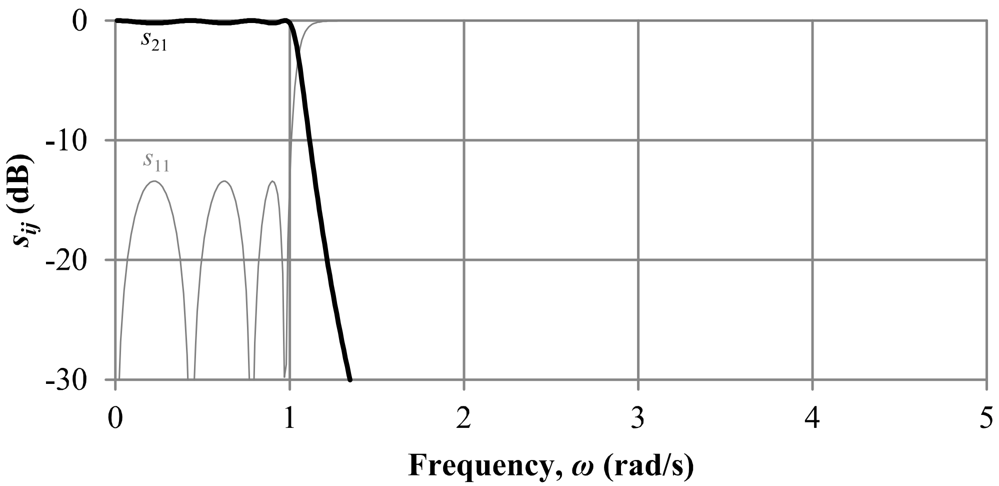


Divide out immittance and corner frequency...

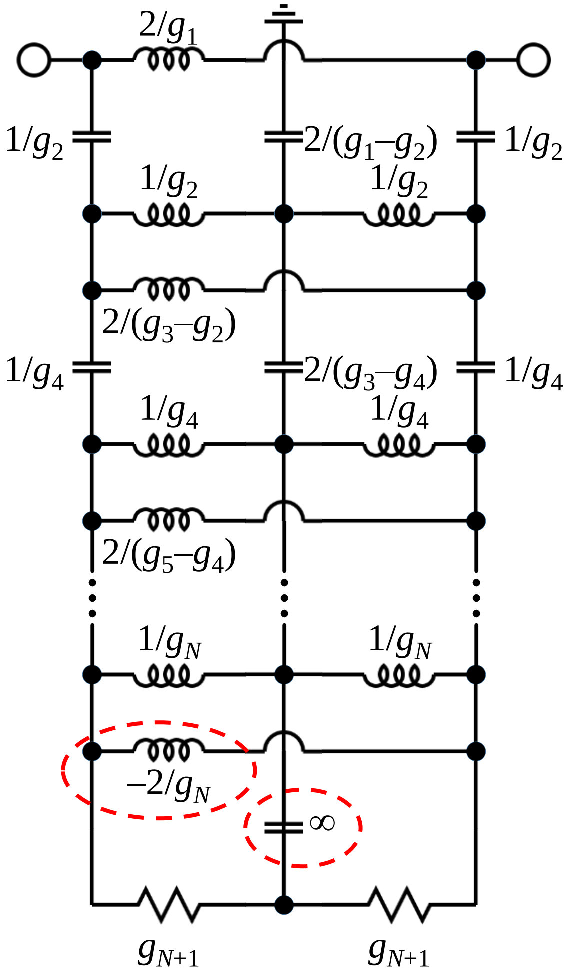
Normalized Ladder Prototype



Normalized Response



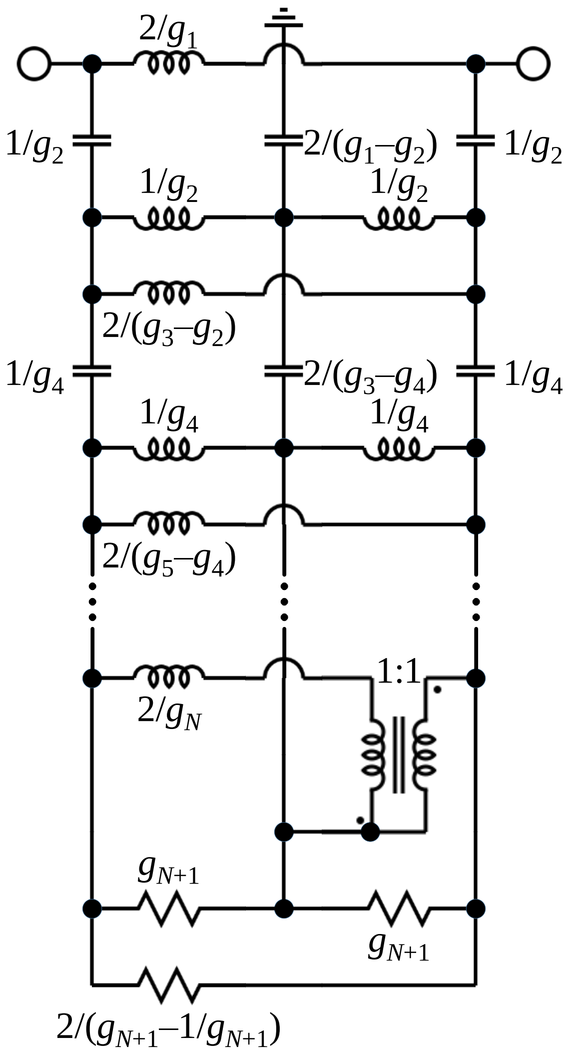
Even-Order Filters



(Element values are normalized for frequency and impedance.)

- One may derive an even-order filter by allowing the last reactive element from an odd-order prototype to become zero.
- This inevitably leaves some negative, zero, or infinitely-valued elements in the bottom-most position.
- The zero/infinite components simply disappear, and the negative elements can be compensated in the usual way.

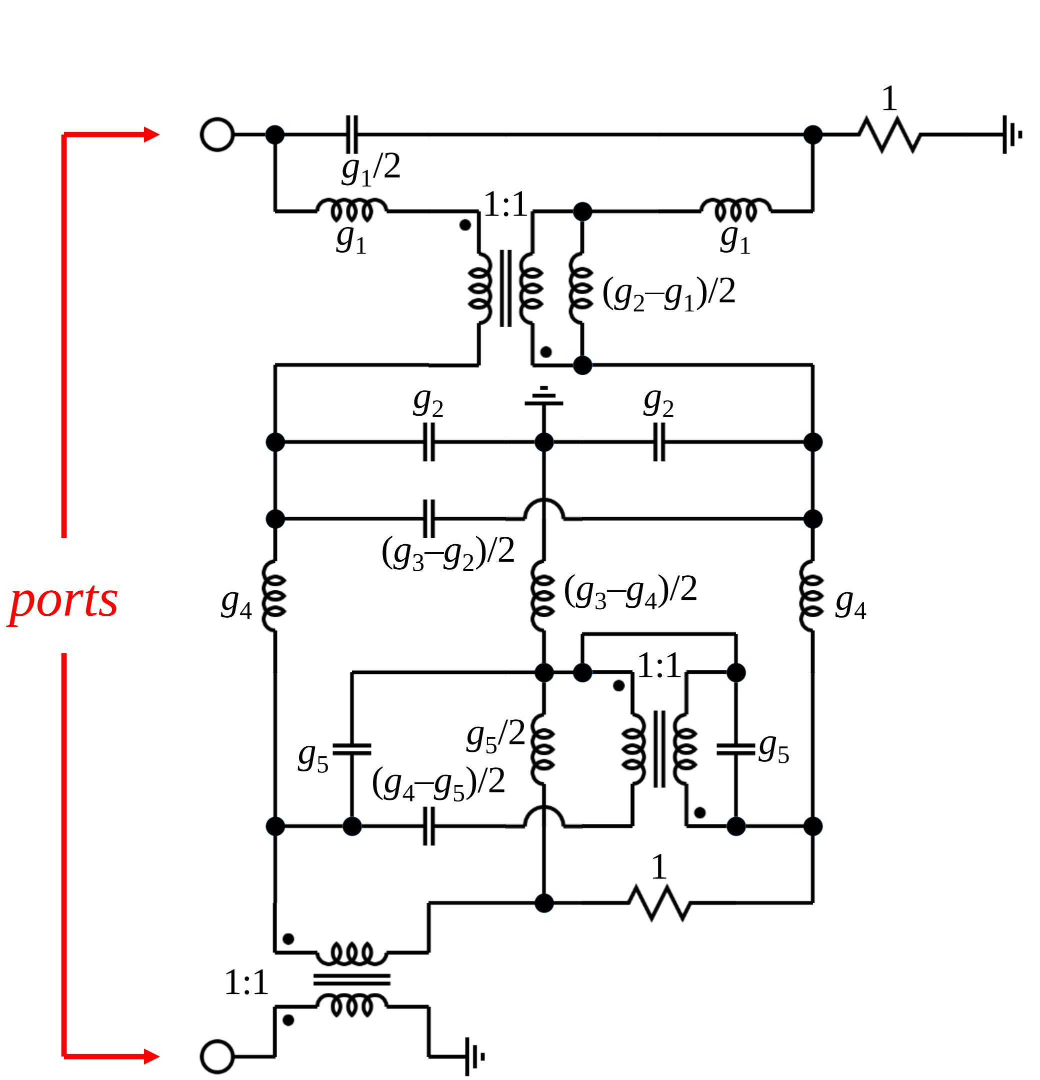
Even-Order Filters



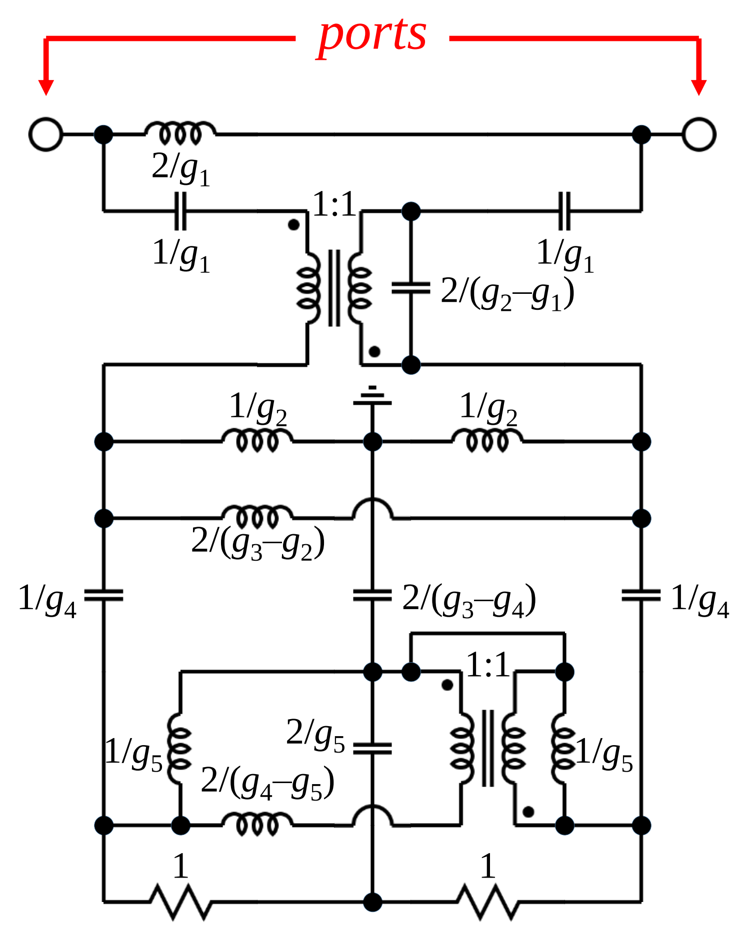
- Many even-order filter prototypes have nonunit-value normalized termination impedance. ($g_{N+1} \neq 1$)
- An extra resistor is then required to meet the duality condition.
- $R = 2/(g_{N+1} – 1/g_{N+1})$
- If this formula gives a negative resistance, than the same transformer identities as before can be used to remove it.

Butterworth is its Own Inverse

“Type-I” Butterworth Topology (N=5)

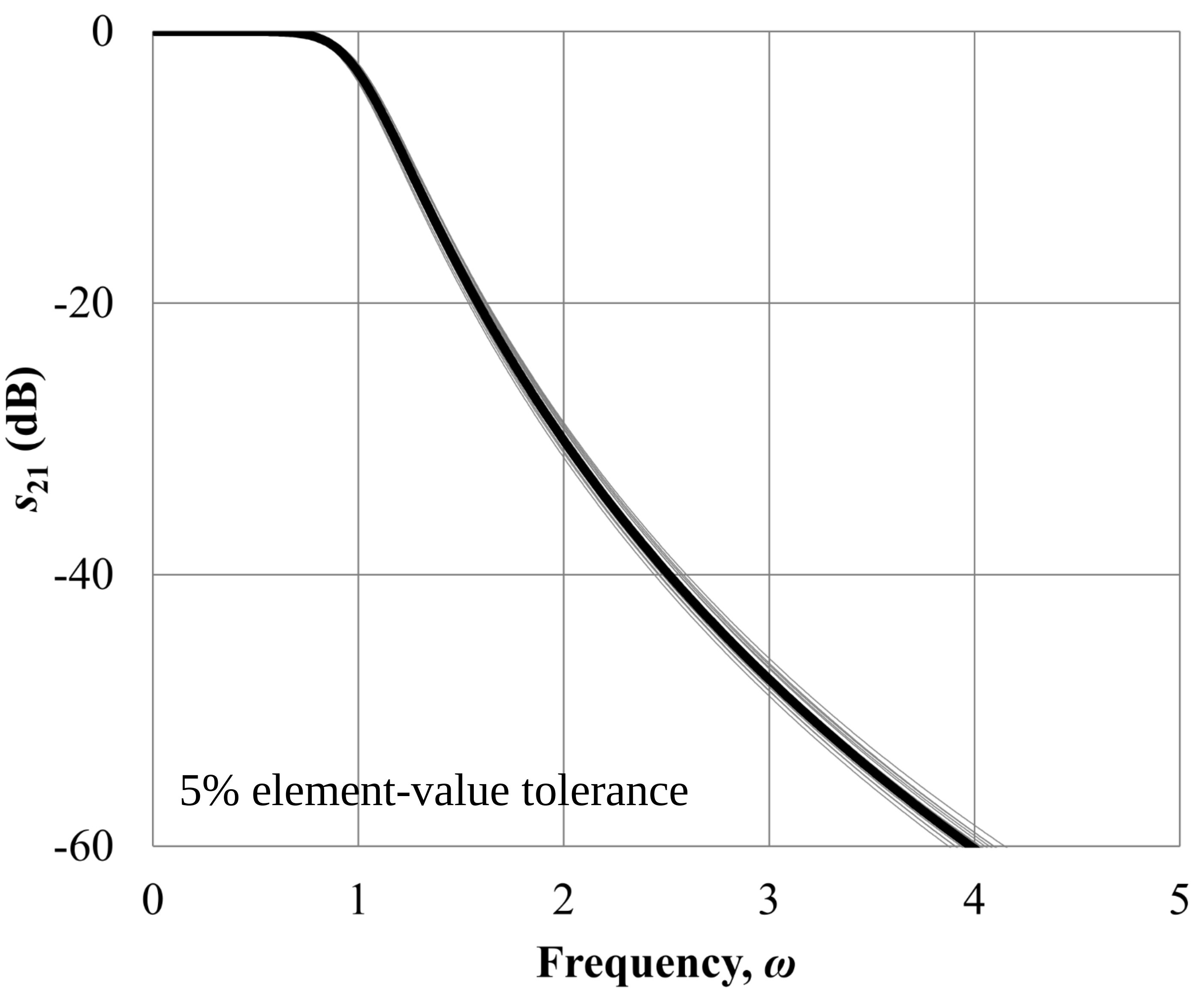


“Type-II” Butterworth Topology (N=5)

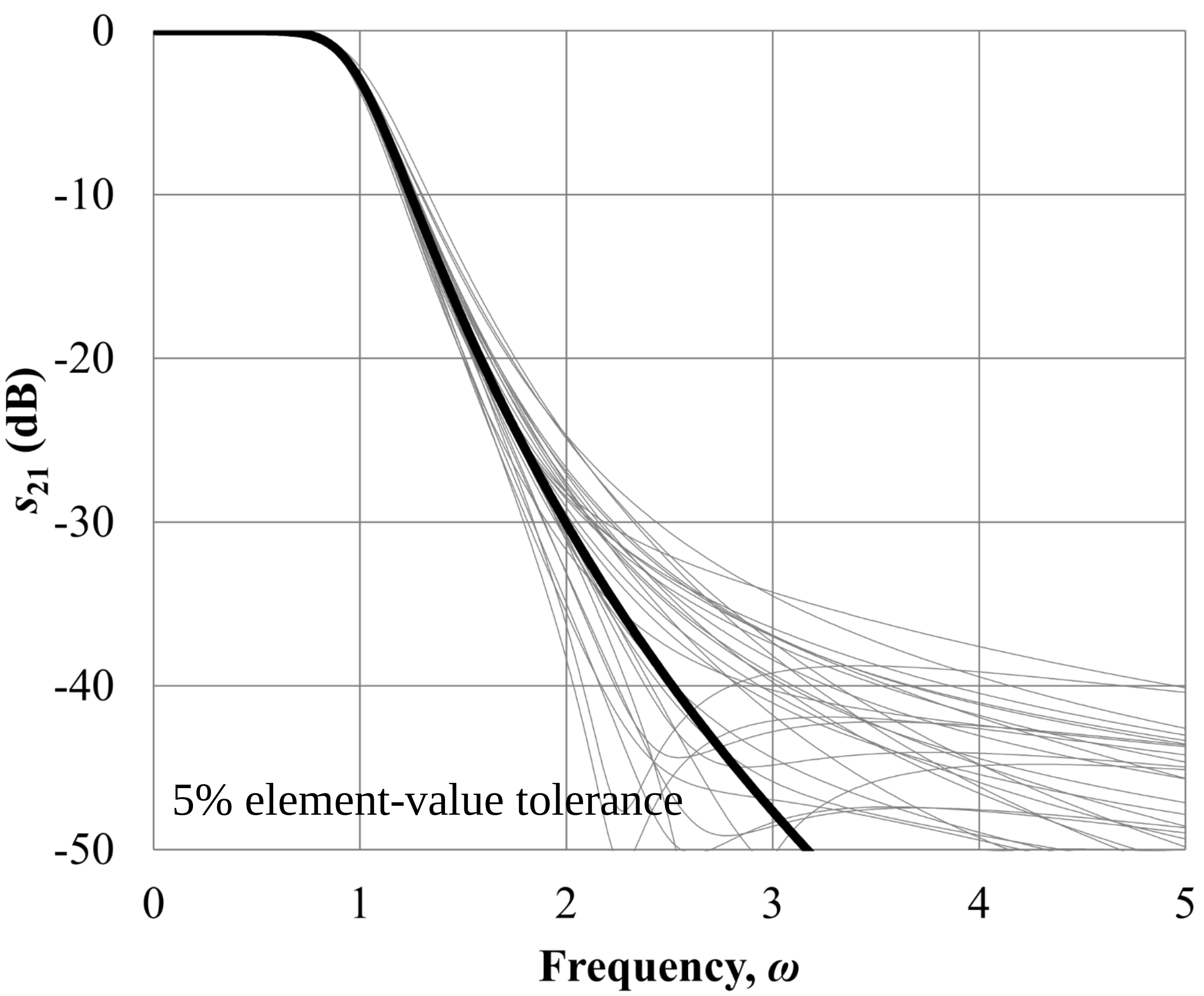


Butterworth is its Own Inverse

“Type-I” Butterworth Topology (N=5)



“Type-II” Butterworth Topology (N=5)

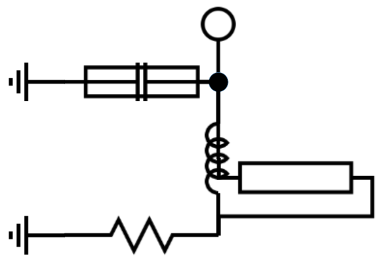


Transmission-Line Topologies

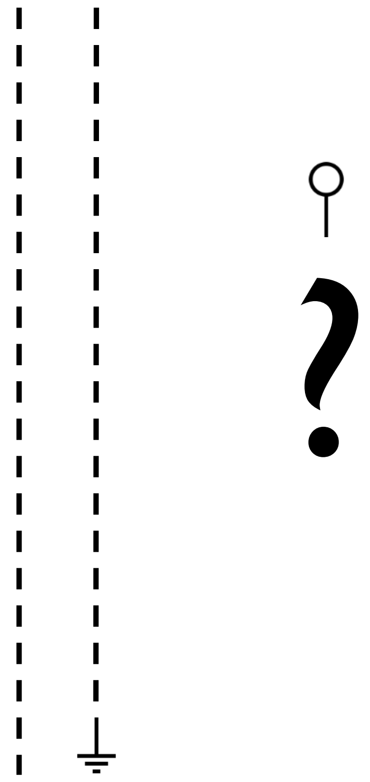
Transmission-Line Implementation

Even-Mode
Equivalent Circuit

Odd-Mode
Equivalent Circuit



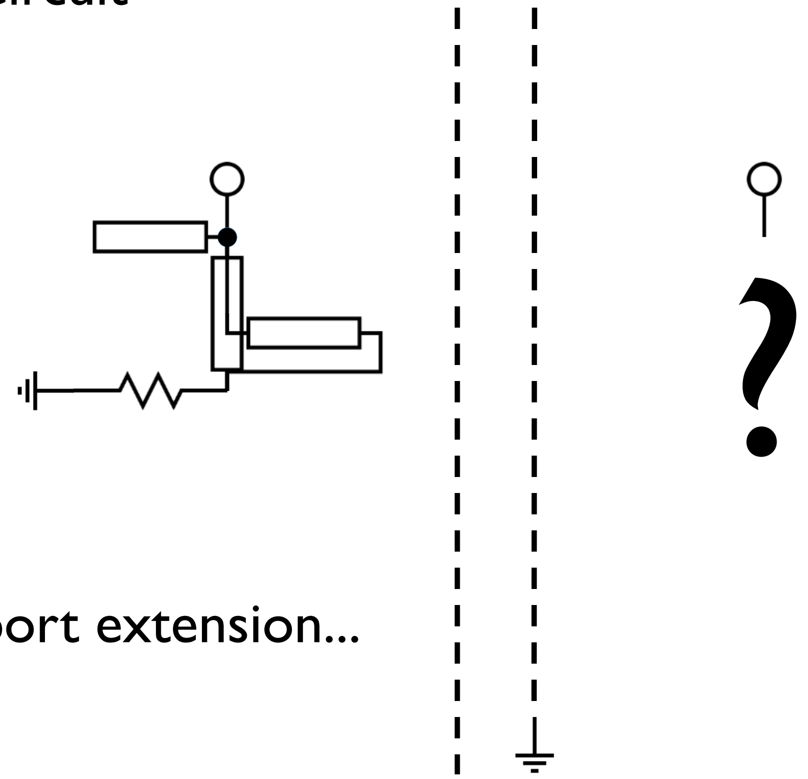
Apply Richard's transformation...



Transmission-Line Implementation

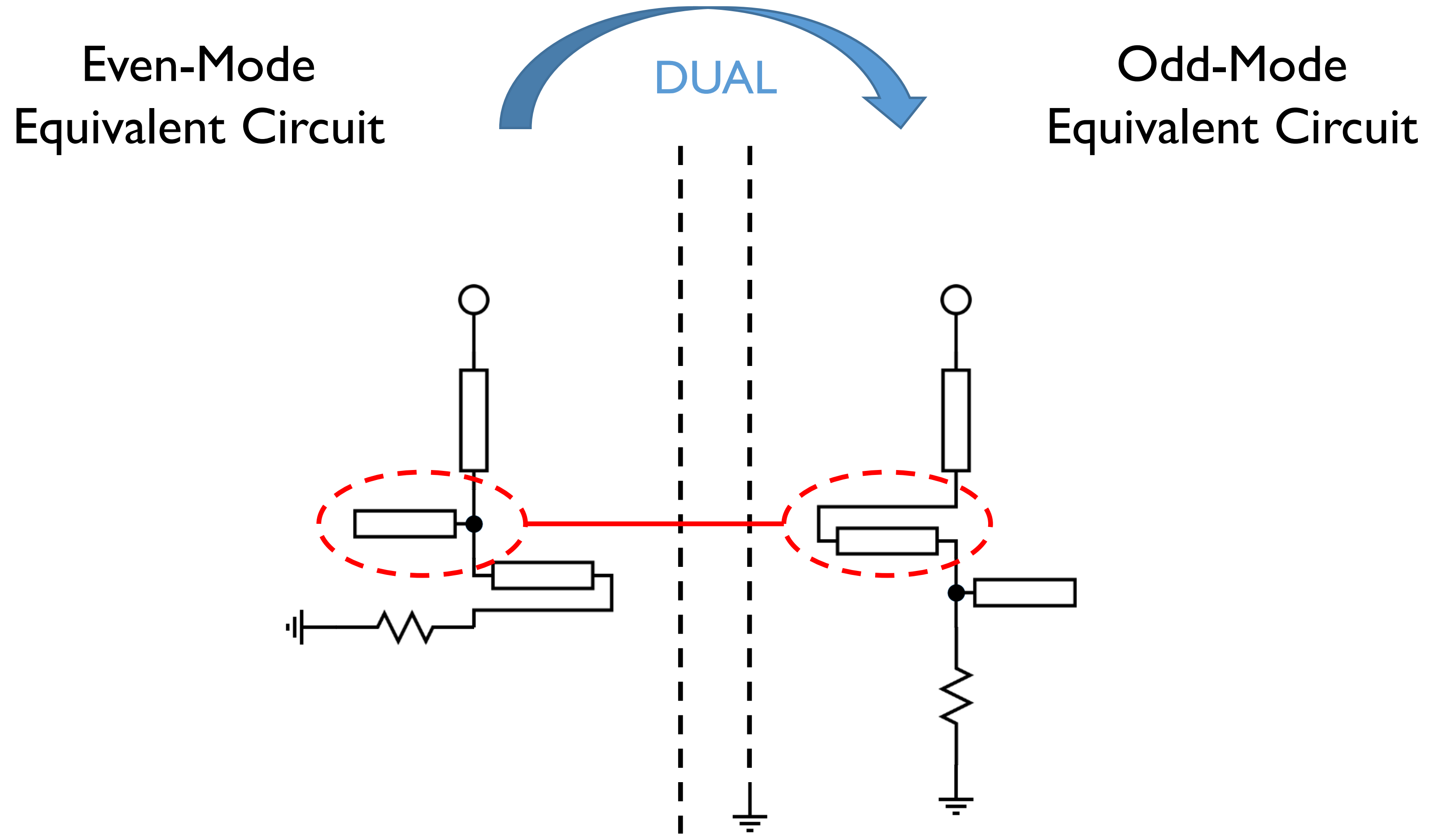
Even-Mode
Equivalent Circuit

Odd-Mode
Equivalent Circuit



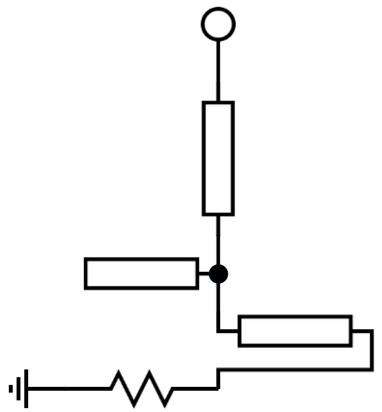
Add port extension…

Transmission-Line Implementation

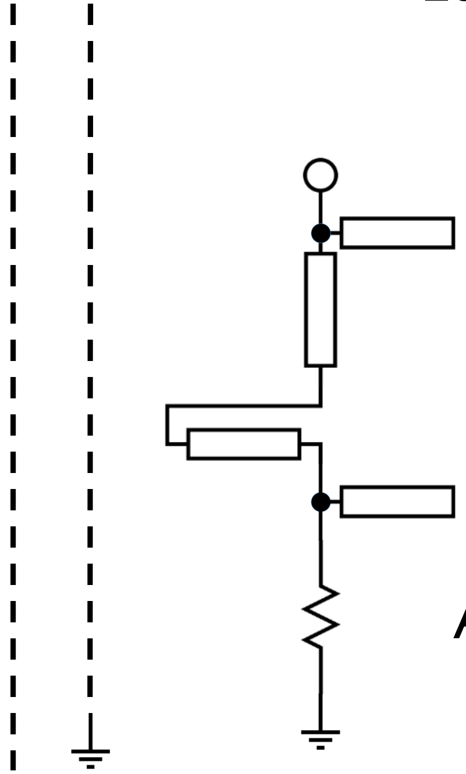


Transmission-Line Implementation

Even-Mode
Equivalent Circuit



Odd-Mode
Equivalent Circuit

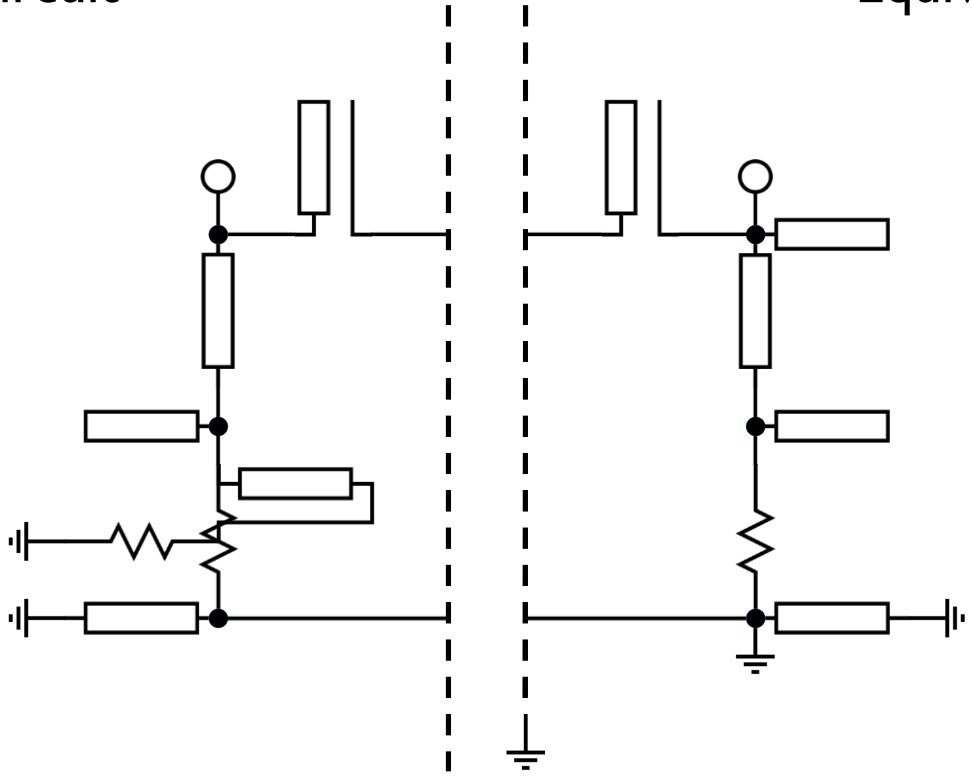


Apply Kuroda's Identity...

Transmission-Line Implementation

Even-Mode
Equivalent Circuit

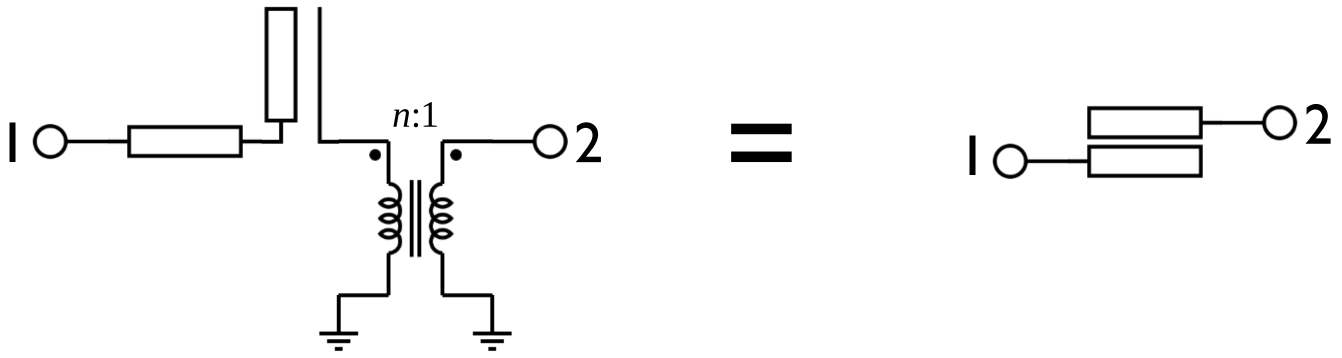
Odd-Mode
Equivalent Circuit



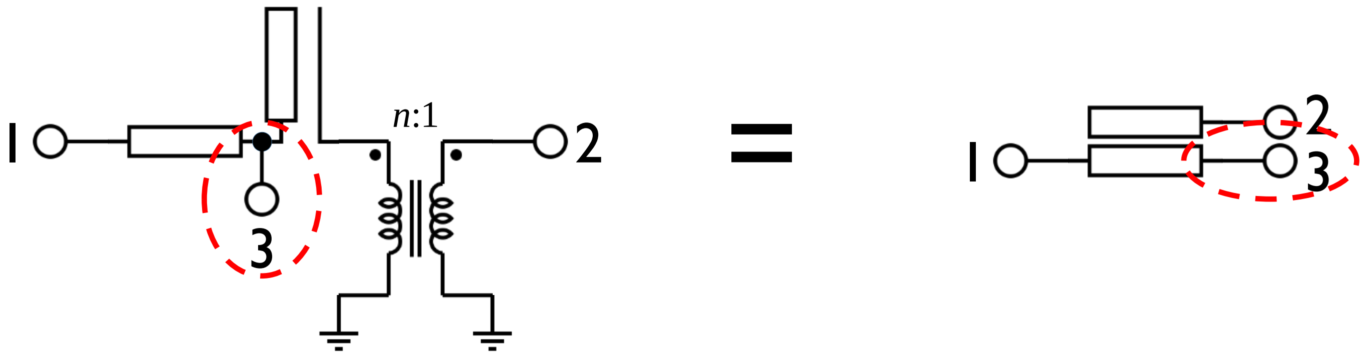
Must restore symmetry...

Coupled-Line Identities

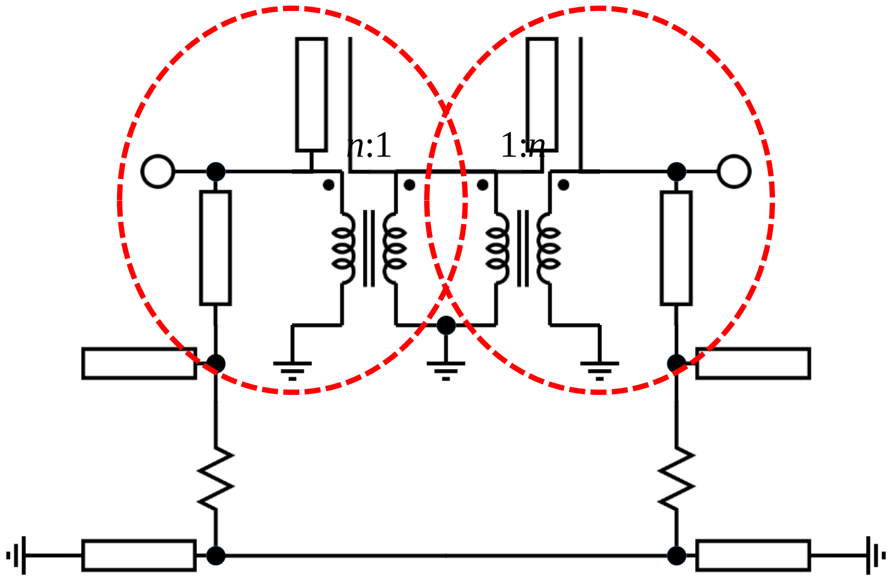
A Well-Known Identity



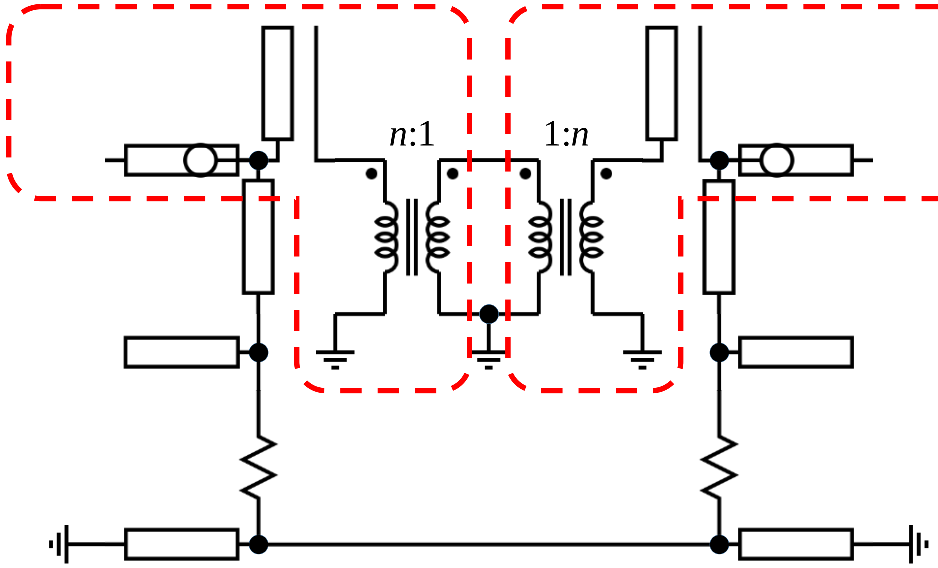
A NOT SO Well-Known Identity



Application of Coupled-Line Identity



OR



Transmission-Line Implementation